Chapter 9

1. You are given that mortality follows the Illustrative Life Table with $i = 0.06$. Assuming that lives are independent and that deaths are uniformly distributed between integral ages, calculate:

   a. $10q_{50:60}$
   b. $10q_{50:60}$
   c. $A_{60:60}$
   d. The net annual premium for a fully discrete joint whole life on (60) and (60) with a death benefit of 1000.
   e. $\overline{A}_{60:60}$
   f. The net annual premium rate for a fully continuous joint whole life on (60) and (60) with a death benefit of 1000.
   g. $10p_{50:60}$
   h. Calculate the probability that the survivor of (50) and (60) dies in year 11.
   i. Calculate the probability that exactly one life of (50) and (60) is alive after 10 years.
   j. $\overline{A}_{60:70}$
   k. The net annual premium rate for a fully discrete survivor whole life on (60) and (70) with a death benefit of 1000. Assume the premium is paid as long as one person is alive.
   l. $\overset{*}{a}_{60:60}$
   m. $\overset{*}{a}_{60:70}$
   n. $\overset{*}{a}_{70:60}$

2. A joint annuity on (50) and (60) pays a benefit of 1 at the beginning of each year if both annuitants are alive. The annuity pays a benefit of $2/3$ at the beginning of each year if one annuitant is alive.

   You are given:

   i. Mortality follows the Illustrative Life Table.
   ii. (50) and (60) are independent lives.
   iii. $i = 0.06$

   Calculate the actuarial present value of this annuity.
3. A joint annuity on (50) and (60) pays a benefit of 1 at the beginning of each year if both
annuitants are alive. The annuity pays a benefit of 2/3 at the beginning of each year if only (50)
is alive. The annuity pays a benefit of ½ at the beginning of each year if only (60) is alive.

You are given:

i. Mortality follows the Illustrative Life Table.
ii. (50) and (60) are independent lives.
iii. \( i = 0.06 \)

Calculate the actuarial present value of this annuity.

4. You are given the following mortality table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( l_x )</th>
<th>( q_x )</th>
<th>( p_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>1000</td>
<td>0.10</td>
<td>0.90</td>
</tr>
<tr>
<td>91</td>
<td>900</td>
<td>0.20</td>
<td>0.80</td>
</tr>
<tr>
<td>92</td>
<td>720</td>
<td>0.40</td>
<td>0.60</td>
</tr>
<tr>
<td>93</td>
<td>432</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>94</td>
<td>216</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>95</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Assume that deaths are uniformly distributed between integral ages and the lives are
independent. Calculate at \( i = 4\% \):

a. \( A_{91:92} \)
b. \( A_{\frac{40:91:3}{90:91:3}} \)
c. \( \ddot{a}_{92:93} \)
5. * You are given:

i. \( 3p_{40} = 0.990 \)

ii. \( 6p_{40} = 0.980 \)

iii. \( 9p_{40} = 0.965 \)

iv. \( 12p_{40} = 0.945 \)

v. \( 15p_{40} = 0.920 \)

vi. \( 18p_{40} = 0.890 \)

For two independent lives aged 40, calculate the probability that the first death occurs after 6 years, but before 12 years.

6. * For a last survivor insurance of 10,000 on independent lives (70) and (80), you are given:

i. The benefit, payable at the end of the year of death, is paid only if the second death occurs during year 5.

ii. Mortality follows the Illustrative Life Table

iii. \( i = 0.03 \)

Calculate the actuarial present value of this insurance.

7. * For a temporary life annuity-immediate on independent lives (30) and (40):

i. Mortality follows the Illustrative Life Table

ii. \( i = 0.06 \)

Calculate \( a_{30:40:10} \)
8. Two lives, \((x)\) and \((y)\), are subject to a common shock model. You are given:

i. \(\mu_x(t) = 0.04\)
ii. \(\mu_y(t) = 0.06\)
iii. \(\mu_x(t)\) and \(\mu_y(t)\) do not reflect the mortality from the common shock.
iv. The mortality from the common shock is a constant force of 0.01.
v. \(\delta = 0.03\)

Calculate:

a. \(tP_x\)

b. \(tP_y\)

c. \(tP_{xy}\)

d. \(e_x\)

e. \(e_y\)

f. \(e_{xy}\)

g. \(\overline{e_{xy}}\)

h. \(\overline{A_{xy}}\)

i. \(\overline{A_{xy}}\)
9. You are given:

i. \( \mu_s(t) = 0.02t \)

ii. \( \mu_s(t) = (40-t)^{-1} \)

iii. The lives are subject to an common shock model with \( \mu = 0.015 \)

iv. \( \mu_s(t) \) and \( \mu_s(t) \) incorporate deaths from the common shock.

Calculate \( 3_q_{xy} \)

10. A male age 65 and a female age 60 are subject a common shock model.

You are given:

i. Male mortality follows the Illustrative Life Table.

ii. Female mortality follows the Illustrative Life Table but for a live five years younger. In other words, the mortality for a female age 60 is equal to the mortality for a person 55 in the Illustrative Life Table.

iii. The above mortality rates do not reflect the common shock risk.

iv. Both lives are subject to a common shock based on a constant force of mortality of 0.01.

v. \( i = 8\% \) - NOTE that it is not 6%.

Calculate:

a. \( 20E_{65} \) where (65) is a male.

b. \( 20E_{60} \) where (60) is a female.

c. \( 20E_{65:60} \) where (65) is a male and (60) is a female.

d. \( 20E_{65:60} \) where (65) is a male and (60) is a female.

11. * You are pricing a special 3-year annuity-due on two independent lives, both age 80. The annuity pays 30,000 if both persons are alive and 20,000 if only one person is alive.

You are given that \( i = 0.05 \) , \( 1p_{80} = 0.91 \) , \( 2p_{80} = 0.82 \) and \( 3p_{80} = 0.72 \).

Calculate the actuarial present value of this annuity.
12. *For a special continuous joint life annuity on \((x)\) and \((y)\), you are given:
   
   i. The annuity payments are 25,000 per year while both are alive and 15,000 per year when only one is alive.
   
   ii. The annuity also pays a death benefit of 30,000 upon the first death.
   
   iii. \(i = 0.06\)
   
   iv. \(\overline{a}_{xy} = 8\)
   
   v. \(\overline{a}_{xy} = 10\)

   Calculate the actuarial present value of this special annuity.

13. Lindsay and Rehan purchase a two year joint term policy. The policy pays a death benefit of 1,000,000 upon the first death. Annual premiums are paid for two years for this policy.

   The probability that Lindsay will die during the next two years is 0.05 and 0.08. The probability that Rehan will die during the next two years is 0.07 and 0.10.

   Lindsay and Rehan are independent lives.

   You are given that \(d = 0.10\).

   Let \(L_0^n\) be the loss at issue random variable for this insurance policy calculated on a net premium basis.

   a. Calculate the annual net benefit premium.

   b. Calculate the standard deviation of \(L_0^n\).
Answers

1. 
   a. 0.26084
   b. 0.03436
   c. 0.47975
   d. 52.20
   e. 0.49400
   f. 56.89
   g. 0.98364
   h. 0.00504
   i. 0.24448
   j. 0.31180
   k. 25.65
   l. 13.0997
   m. 12.1584
   n. 3.5891

2. 12.8768
3. 12.7182

4. 
   a. 0.93867
   b. 0.83045
   c. 1.28846

5. 0.067375
6. 234.82
7. 7.1687

8. 
   a. $e^{-0.05t}$
   b. $e^{-0.07t}$
   c. $e^{-0.11t}$
   d. 20
   e. 14.286
   f. 9.091
   g. 25.195
   h. 0.78571
   i. 0.53929

9. 0.06994
10.
   a. 0.05498
   b. 0.10970
   c. 0.03434
   d. 0.13034

11. 80,431.70

12. 246,015.46

13.
   a. 126,975.03
   b. 401,700