1. Purdue Life Insurance Company (PLIC) wants to develop a multiple decrement model to use in pricing a two year term insurance policy. The multiple decrement model will have two decrements – (d) death and (w) lapse.

However, PLIC does not have sufficient data to develop a multiple decrement table. Therefore, PLIC will use an independent mortality table and an independent lapse table to develop a multiple decrement table.

You are given the following independent mortality and lapse rates for age 80:

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>q_x^(d)</th>
<th></th>
<th>x</th>
<th>q_x^(w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.10</td>
<td></td>
<td>80</td>
<td>0.30</td>
<td></td>
</tr>
</tbody>
</table>

In deriving the multiple decrement table, you assume that decrements are uniformly distributed in the multiple decrement table.

a. (6 points) Complete the following multiple decrement table. Be sure to show your work:

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>l_x^v</th>
<th>d_x^(d)</th>
<th>d_x^(w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>10,000</td>
<td>843.731579</td>
<td>2856.268421</td>
<td></td>
</tr>
<tr>
<td>81</td>
<td>6300</td>
<td>800</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>82</td>
<td>5000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ p_{x0}^{(r)} = (1 - 0.1)(1 - 0.3) = 0.63 \]

\[ q_{x0}^{(c)} = 1 - 0.63 = 0.37 \]

\[ l_{x1}^{(c)} = 10,000(0.63) = 6300 \]

\[ d_{x1}^{(c)} = (6300 - 5000) - 500 = 800 \]

\[ q_{x0}^{(d)} = 1 - p_{x0}^{(r)} q_{x0}^{(w)} \]

\[ = 0.37 \]

\[ q_{x0}^{(d)} = \frac{\ln(0.9^{0.37})}{\ln(0.63)} = 0.084373158 \]

\[ d_{x0}^{(d)} = 10,000(0.084373158) = 843.731579 \]

\[ d_{x0}^{(w)} = (10,000 - 6300) - 843.731579 = 2856.265421 \]
PLIC wants to develop the premium for a fully discrete two year term insurance policy on (80). The death benefit is 250,000. If the insured has not lapsed or withdrawn at the end of two years, the premium will be refunded without interest. In other words, two times the premium will be paid. If the policy is surrendered in either year, there is no benefit paid on withdrawal.

b. (8 points) Using the above multiple decrement table and \( i = 10\% \), calculate the net benefit premium.

\[
P(10,000 \times 6300v) = 250,000(843.731579 + 800v^2) + 2P(5000v^2)
\]

\[
P(10,000 + 6300v - 10,000v^2) = 250,000(843.731579v + 800v^2)
\]

\[P = 47,843.43\]

(6 points) Calculate the reserve for this policy at the end of year 1

\[
V = PVB - PVP = \frac{800}{6300} (250,000v) + \frac{5000}{6300} (2)(47,843.43) - 47,843.43 = 50,054.74
\]