1. A life insurance company sells a whole life policy to (95) with a death benefit of 1,000,000.

You are given that \( q_{95} = 0.40 \), \( q_{96} = 0.70 \) and \( q_{97} = 1.00 \).

You are also given that the current spot interest rate curve:

<table>
<thead>
<tr>
<th>Time ( t )</th>
<th>( y_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
</tr>
<tr>
<td>3</td>
<td>0.06</td>
</tr>
<tr>
<td>4</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Premiums are paid annually.

(4 points) The net benefit premium is 527,000 to the nearest 1000. Calculate the net benefit premium to the nearest 1.

Solution:

Let's calculate \( l's \). \( l_{95} = 100 \); \( l_{96} = 100(1 - 0.4) = 60 \); \( l_{97} = 60(1 - 0.7) = 18 \); \( l_{98} = 0 \)

\[
P[100 + 60(1.04)^{-1} + 18(1.05)^{-2}] = (1,000,000)[40(1.04)^{-1} + 42(1.05)^{-2} + 18(1.06)^{-3}]\]

\[\therefore P = 526,781.61\]

(7 points) Calculate the net benefit reserve at the end of one year and at the end of two years.

Solution:
This can be done using the recursive formula or PVFB-PVFP. Either way, you must use forward interest rates when applicable.

\[ V_0 = 0 \]
\[ V_1 = \frac{(V_0 + P)(1 + f(0,1)) - (S)(q_x)}{1 - q_x} = \frac{(0 + 526,781.61)(1.04) - (1,000,000)(0.4)}{1 - 0.4} = 246,421.46 \]

\[ V_2 = \frac{(V_1 + P)(1 + f(1,2)) - (S)(q_{x+1})}{1 - q_{x+1}} = \frac{(246,421.46 + 526,781.61)(1.060096154) - (1,000,000)(0.7)}{1 - 0.7} = 398,898.66 \]

**Or**

\[ V_1 = 1,000,000 \left[ \frac{42}{60} (1 + f(1,2))^{-1} + \frac{18}{60} (1 + f(1,3))^{-2} \right] - (526,781.61) \left[ 1 + \frac{18}{60} (1 + f(1,2))^{-1} \right] \]

\[ 1 + f(1,3) = \left[ \frac{(1.06)^3}{1.04} \right]^{1/2} = 1.070143772 \]

\[ = 1,000,000 \left[ \frac{42}{60} (1.060096154)^{-1} + \frac{18}{60} (1.070143772)^{-2} \right] - (526,781.61) \left[ 1 + \frac{18}{60} (1.060096154)^{-1} \right] \]

\[ = 246,421.45 \]

\[ V_2 = 1,000,000(1 + f(2,3))^{-1} - 526,781.61 \]

\[ 1 + f(2,3) = \left[ \frac{(1.06)^3}{(1.05)^2} \right] = 1.080286621 \]

\[ = 1,000,000(1.080286621)^{-1} - 526,781.61 \]

\[ = 398,898.65 \]
(10 points) Let \( L_0^n \) be the loss at issue random variable for this insurance. As we discussed in class, since we are using spot interest rates, we cannot calculate the \( \text{Var}[L_0^n] \) using the standard formula of \( \text{Var}[L_0^n] = \left( S + \frac{P}{d} \right)^2 \left( A_d - [A_d]^2 \right) \). We must calculate it from first principles. Calculate the \( \text{Var}[L_0^n] \).

**Solution:**

<table>
<thead>
<tr>
<th>Event</th>
<th>Present Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Die Year 1</td>
<td>(1,000,000)(1.04)(^1)-526,781.61=434,756.85</td>
<td>0.40</td>
</tr>
<tr>
<td>Die Year 2</td>
<td>(1,000,000)(1.05)(^2)-526,781.61(1+1.04(^{-1}))=-126,272.91</td>
<td>0.42</td>
</tr>
<tr>
<td>Die Year 3</td>
<td>(1,000,000)(1.06)(^3)-526,781.61(1+1.04(^{-1})+1.05(^{-2}))=-671,489.55</td>
<td>0.18</td>
</tr>
</tbody>
</table>

\[
\text{Var}[L_0^n] = E[L_0^{n^2}] - \left( E[L_0^n] \right)^2 = (434,756.85)^2(0.4) + (-126,272.91)^2(0.42) + (-671,489.55)^2(0.18) - 0
\]

\[
= 163,463,922,400 = (404,306.72)^2
\]