STAT 490
Spring 2016
Test 3

April 21, 2016

1. Jack and Tania who are business partners purchase a survivor whole life policy which pays a death benefit of 500,000 at the end of the year of the second death. The premiums for this policy are only payable if both Jack and Tania are alive.

   Jack and Tania are both age 30. You further assume that Jack and Tania have independent lifetimes.

   You are given that mortality follows the Illustrative Life Table with interest at an annual effective rate of 6%.

   Calculate the net benefit premium for this policy.

   Solution:

$$P\ddot{a}_{30:30} = (500,000)A_{\overline{30:30}}$$

$$P(14.9901) = (500,000)(A_{30} + A_{30} - A_{30:30}) = (500,000)(2 \cdot 0.10248 - 0.15150)$$

$$P = \frac{(500,000)(0.05346)}{14.9901} = 1783.18$$

2. A participating whole life insurance policy is issued on (50). The policy pays a death benefit of 250,000 at the end of the year of death. You are given:

- The gross premium is 5600.
- The following reserves:

${}_{14}V$	58,700
${}_{15}V$	63,500
${}_{16}V$	68,500

- Actual mortality experienced is 80% of the Illustrative Life Table.
- Actual interest earned is 6.5%
- Expenses for the policy are:
  - Issue expenses of 200 per policy and 40% of premium.
  - Recurring expenses are 50 per policy and 6% of premium in all years including the first year.
- Cash values are 85% of the reserves.
- Withdrawals will occur at the end of the year. The rate of withdrawals will be 20% in the first year, 15% in the second year, and 10% per year thereafter.
- 75% of the profit each year will be distributed to the policyholders who were in force at the start of the year.

Calculate the dividend in for the 15th year.

Solution:

$$P_t = 5600$$
$${}_{14}V = 58,700$$
$$e_{15} = (5600)(0.06) + 50 = 386.00$$
$$Int = (58,700 + 5600 - 386)(0.065) = 4154.41$$
$$DeathBenefits = (250,000)(0.80)(0.01952) = 3904.00$$
$$WithdrawalBenefits = (63,500)(0.85)(1-(0.80)(0.01952))(0.1) = 5313.21$$
$$End\ \mathrm{Re}serve = (63,500)(1-(0.80)(0.01952))(1-0.1) = 56,257.55$$

$$\text{Profit} = 5600 + 58,700 - 386.00 + 4154.41 - 3904.00 - 5313.21 - 56,257.55 = 2593.65$$

$$Div = (2593.65)(0.75) = 1945.24$$

3. You are given:

   a. $q_x^{(1)} = 0.05$
   b. $q_x^{(2)} = 0.15$
   c. Each decrement is uniformly distributed in the multiple decrement table.

   Calculate $q_x'^{(2)}$.

   Solution:

$$p_x'^{(2)} = \left[p_x^{(\tau)} \right]^{\frac{q_x^{(2)}}{q_x^{(\tau)}}} = \left[1 - 0.05 - 0.15\right]^{\frac{0.15}{0.05+0.15}} = 0.845897011$$

$$q_x'^{(2)} = 1 - p_x'^{(2)} = 1 - 0.845897011 = 0.154103$$

4. Scott works for Cunningham Actuarial Consultants. He began working at Cunningham on his 40th birthday. Today is Scott's 50th birthday. During the last 10 years Scott has earned a total salary of $1,000,000. Scott's salary for the next year will be $125,000.

   Cunningham has a defined benefit pension plan that pays 3% of total career average earnings for each year of service. Assume that Scott continues to work until his 65th birthday and that he will receive an increase in salary of 5% on each birthday.

   At age 65, Scott will be entitled to an annual pension benefit of 110,000 to the nearest 5000. Calculate the annual pension benefit to the nearest 1 .

   Solution:

$$Benefit = (Rate)(NumberOfYears)\frac{CareerEarnings}{NumberOfYears} = (Rate)(CareerEarnings) =$$

$$(0.03)(1,000,000 + 125,000(1 + (1.05 + 1.05^2 + \ldots + 1.05^{14}) =$$

$$(0.03)\left(1,000,000 + 125,000\left[\frac{1.05^{15} - 1}{0.05}\right]\right) = 110,919.61$$

   Scott's benefit will be paid as a survivor annuity beginning on Scott's 65th birthday. The survivor annuity will pay the amount determined above at the beginning of each year as long as Scott and his wife are alive. If Scott is not alive, but his wife is alive, the annuity will pay 40% of the above benefit at the beginning of each year. If Scott is alive, but his wife is not, the annuity will pay 50% of the above benefit at the beginning of each year.

   For determining annuity values, you are given that mortality follows the Illustrative Life Table and the annual effective interest rate is 6%. You are also given that Scott's wife is 10 years younger than Scott.

   Calculate the amount of money that the Cunningham needs when Scott is age 65 to buy this annuity.

   Solution:

$a =$ amount if Scott alive = 0.5 and b=amount if wifeis alive = 0.4==>a+b+c=1==>c=0.1

$$PV = (110,919.61)(0.5\ddot{a}_{65} + 0.4\ddot{a}_{55} + 0.1\ddot{a}_{55:65}) =$$

$$(110,919.61)\left[0.5(9.8969) + 0.4(12.2758) + 0.1(8.8966)\right] = 1,192,211.70$$

5. You are given the following information for a Type A universal life policy:

Policy Year	Annual Premium	Percent of Premium Charge	Annual Expense Charge	Total Death Benefit	Annual Cost of Insurance Charge	Annual Discount Rate for COI	Annual Credited Interest Rate
1	10,000	10%	75	100,000	0.025	4%	6.5%

Calculate the account value at the end of the first year.

Solution:

$$COI = \frac{v_q \cdot q_{x+t-1}\left[DB - (AV_{t-1} + P(1-f) - e)(1+i^c)\right]}{1 - v_q \cdot q_{x+t-1}(1+i^c)} =$$

$$\frac{(1.04)^{-1}(0.025)\left[100,000 - (0 + 10,000(1-0.1) - 75)(1.065)\right]}{1-(1.04)^{-1}(0.025)(1.065)} = 2232.51$$

$$AV_1 = (AV_0 + P(1-f) - e - COI)(1+i^c) =$$

$$(0 + 10,000(1-0.1) - 75 - 2232.51)(1.065) = 7127.50$$

6. **THIS PROBLEM IS CONTINUED ON THE NEXT PAGE.**

Queenie who is (x) purchases a two year term insurance which pays a death benefit of 300,000 at the end of the year of death.

You are given:

a. $q_{x+t} = 0.2 + 0.1t$
b. The one year spot rate is 8%.
c. The two year spot rate is uncertain and has the following distribution:
   i. 10% with a 70% probability
   ii. 12% with a 30% probability
d. The gross annual premium for this policy is 80,000.
e. The policy is subject to the following expenses:
   i. Commissions of 20% in the first year and 10% in the second year.
   ii. Per policy expense of 100 each year at the start of the year.

L_0^g is the loss at issue random variable on a gross premium basis.

Calculate $E[L_0^g]$.

Solution:

Case 1 ==> Die Year 1==>Prob=0.2

$$L_0^g = (300,000)(1.08)^{-1} + 100 - (80,000)(0.8) = 213,877.78$$

Case 2 ==> Die Year 2 and 2 Year Spot is 10% ==>Prob=(0.8)(0.3)(0.7)=0.168

$$L_0^g = (300,000)(1.10)^{-2} + 100 + 100(1.08)^{-1} - (80,000)(0.8) - (80,000)(0.9)(1.08)^{-1} = 117,459.81$$

Case 3 ==> Die Year 2 and 2 Year Spot is 12% ==>Prob=(0.8)(0.3)(0.3)=0.072

$$L_0^g = (300,000)(1.12)^{-2} + 100 + 100(1.08)^{-1} - (80,000)(0.8) - (80,000)(0.9)(1.08)^{-1} = 108,684.09$$

Case 4 ==> Live 2 Years ==>Prob=(0.8)(0.7)=0.56

$$L_0^g = 100 + 100(1.08)^{-1} - (80,000)(0.8) - (80,000)(0.9)(1.08)^{-1} = -130,474.07$$

$$E[L_0^g] = (0.2)(213,877.78) + (0.168)(117,459.81) + (0.072)(108,684.09) + (0.56)(-130,474.07)$$

$$= -2371.42$$

Calculate $Var[L_0^g]$.

Solution:

$$Var[L_0^g] = E[(L_0^g)^2] - \left(E[L_0^g]\right)^2$$

$$E[(L_0^g)^2] = (0.2)(213,877.78)^2 + (0.168)(117,459.81)^2 + (0.072)(108,684.09)^2 + (0.56)(-130,474.07)^2$$

$$Var[L_0^g] = (0.2)(213,877.78)^2 + (0.168)(117,459.81)^2 + (0.072)(108,684.09)^2 + (0.56)(-130,474.07)^2 -$$

$$(-2371.42)^2 = 21,844,612,000$$