1. You are given that mortality follows the mortality table above. You are also given that $i = 0.05$.

$Z$ is the present value random variable for a whole life insurance policy issued to (75) which pays a death benefit of 1000 at the end of the year of death.

Determine the $\text{Var}[Z]$.

2. Wang Insurance Company issues 900 policies which provide term insurance coverage until age 90. The lives covered by these 900 policies are independent lives. Each policy is sold to (50) and provides a death benefit of 25,000 payable at the end of the year of death.

You are given:
   a. Mortality follows the Illustrative Life Table.
   b. $i = 6\%$

Assuming the normal distribution, calculate the amount that Wang must have on hand at time 0 to be 97.5% certain that the company can cover the future death benefits.

3. You are given:
   a. $d = 0.10$
   b. $q_{50} = 0.05$
   c. $Z_{90}$ is the present value random variable for a whole life insurance on (90) with a death benefit 1000 paid at the end of the year of death.
   d. $Z_{91}$ is the present value random variable for a whole life insurance on (91) with a death benefit 1000 paid at the end of the year of death.
   e. $E[Z_{90}] = 800$
   f. $\text{Var}[Z_{90}] = 60,000$

Calculate $\text{Var}[Z_{91}]$. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$l_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>2000</td>
</tr>
<tr>
<td>76</td>
<td>1600</td>
</tr>
<tr>
<td>77</td>
<td>960</td>
</tr>
<tr>
<td>78</td>
<td>480</td>
</tr>
<tr>
<td>79</td>
<td>120</td>
</tr>
<tr>
<td>80</td>
<td>0</td>
</tr>
</tbody>
</table>
4. $Z$ is the present value random variable for a whole life on (77) with a death benefit of 1000 payable at the end of the year of death.

You are given:

   a. Mortality follows:

   \[
   \begin{array}{|c|c|}
   \hline
   x & l_x \\
   \hline
   75 & 2000 \\
   76 & 1600 \\
   77 & 960 \\
   78 & 480 \\
   79 & 120 \\
   80 & 0 \\
   \hline
   \end{array}
   \]

   b. $v = 0.9$

   Calculate $Var[Z]$.

5. $Y$ is the present value random variable for a temporary 10 year life annuity due to (80) which pays 1000 annually.

You are given:

   a. Mortality follows the Illustrative Life Table
   
   b. $i = 6\%$

   Calculate the $Var(Y)$

6. $Y$ is the present value random variable for a whole life annuity payable to \(x\) with continuous payments at an annual rate of 1.

You are given:

   a. $1000\, \overline{A}_x = 800.00$
   
   b. $1000 \cdot 2\, \overline{A}_x = 652.50$
   
   c. $Var[Y] = 5$
   
   d. Deaths are uniformly distributed between integer ages.

   Calculate $\ddot{a}_i$. 

April 12, 2016
Copyright Jeffrey Beckley 2015, 2016
7. You are given:

i. Mortality follows

<table>
<thead>
<tr>
<th>( x )</th>
<th>( l_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>2000</td>
</tr>
<tr>
<td>76</td>
<td>1600</td>
</tr>
<tr>
<td>77</td>
<td>960</td>
</tr>
<tr>
<td>78</td>
<td>480</td>
</tr>
<tr>
<td>79</td>
<td>120</td>
</tr>
<tr>
<td>80</td>
<td>0</td>
</tr>
</tbody>
</table>

ii. \( v = 0.90 \)

iii. \( Y \) is the present value random variable for a 2 year temporary life annuity to (75) with annual payments of 1 at the beginning of the year.

Calculate the \( \text{Var}[Y] \).

8. A temporary life annuity immediate pays 10,000 at the end of one year if (80) is alive and 20,000 at the end of two years if (80) is alive. \( Y \) is the present value random variable for this annuity.

You are given:

i. \( q_{80} = 0.05 \)

ii. \( q_{81} = 0.08 \)

iii. \( d = 10\% \)

Calculate the \( \text{Var}[Y] \).
9. (Written Answer Question) A temporary life annuity immediate pays 10,000 at the end of one year if (80) is alive and 20,000 at the end of two years if (80) is alive. $L^n_0$ is the present value loss at issue random variable for this annuity using the net benefit premium.

You are given:

iv. $q_{80} = 0.04$

v. $q_{81} = 0.07$

vi. $i = 10\%$

The annuity is purchased with a single net benefit premium payable at the beginning of the annuity.

a. Your boss estimates the single net benefit premium to be 23,500. He is accurate to the nearest 100. Calculate the net annual premium to the nearest 1.

b. Write an expression (or expressions) for the loss at issue present value random variable. Include any constraints on each expression.

c. Calculate the $Var[L^n_0]$.

10. Rafidah purchases a fully continuous whole life insurance policy with a death benefit of 50,000 payable at the moment of death. Level net premiums are paid continuously for as long as Rafidah is alive.

You are given:

a. Rafidah is now age 50.

b. Mortality follows the Illustrative Life Table

c. $i = 6\%$

d. Deaths are uniformly distributed between integer ages.

Calculate the Variance for the loss random variable, $L^n_0$, for this policy.
11. You are given the following mortality table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$l_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>1000</td>
</tr>
<tr>
<td>91</td>
<td>900</td>
</tr>
<tr>
<td>92</td>
<td>700</td>
</tr>
<tr>
<td>93</td>
<td>400</td>
</tr>
<tr>
<td>94</td>
<td>100</td>
</tr>
<tr>
<td>95</td>
<td>0</td>
</tr>
</tbody>
</table>

For a whole life to (92) with a death benefit of 50,000 payable at the end of the year of death and level annual premiums, the expenses are 400 per policy at issue and 50 per policy at the beginning of each year including the first year.

You are given that $i = 5\%$.

a. Calculate the level gross premium using the equivalence principle.

b. Complete the following table:

<table>
<thead>
<tr>
<th>$K_x$</th>
<th>$l^g_0$</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. Calculate the variance of the loss at issue random variable.

d. Calculate the expected value and the variance of the loss at issue random variable if the gross premium was 30,000.
12. Surin purchases a fully discrete 2 year term insurance with a death benefit of 10,000. Surin is (80).

You are given that:

i. $q_{80} = 0.08$

ii. $q_{81} = 0.12$

iii. $v = 0.9$

$L_0^n$ is the loss-at-issue random variable based on the net premium.

Calculate the $\text{Var}[L_0^n]$.

13. For a fully discrete whole life of 100 on (65), you are given:

iv. Maintenance expenses are 2.50 per policy at the start of each year

v. Issue expenses at the start of the first policy year of 1.

vi. Mortality follows the Illustrative Life Table.

vii. $i = 6\%$

The gross premium is determined using the equivalence principle.

Calculate the variance of the loss-at-issue random variable.