(1) **Review of Algebra:**

(a) Equalities, inequalities, absolute values

(b) Distance between $P(x_1, y_1)$ and $P(x_2, y_2)$ is $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(c) Equations of lines:
   
   (i) Point-Slope Form: $y - y_1 = m(x - x_1)$

   (ii) Slope-Intercept Form: $y = mx + b$

(d) $L_1 \parallel L_2 \iff m_1 = m_2$; $L_1 \perp L_2 \iff m_2 = -\frac{1}{m_1}$

(e) Equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$.

(2) **Review of Trigonometry:**

(a) Length of arc subtended by $\theta$: $a = r\theta$

(b) Definition of trig functions ($\cos \theta, \sin \theta, \tan \theta, \csc \theta, \sec \theta, \cot \theta$); basic trig values of well-know angles.

(c) Basic trig identities (including):

   (i) \[
   \begin{align*}
   \sin^2 \theta + \cos^2 \theta &= 1 \\
   \tan^2 \theta + 1 &= \sec^2 \theta
   \end{align*}
   \]

   (ii) \[
   \begin{align*}
   \sin(-\theta) &= -\sin \theta \\
   \cos(-\theta) &= \cos \theta
   \end{align*}
   \]

   (iii) \[
   \begin{align*}
   \sin(\theta + 2\pi) &= \sin \theta \\
   \cos(\theta + 2\pi) &= \cos \theta
   \end{align*}
   \]

   (iv) \[
   \begin{align*}
   \sin 2\theta &= 2\sin \theta \cos \theta \\
   \cos 2\theta &= \cos^2 \theta - \sin^2 \theta
   \end{align*}
   \]

   (v) \[
   \begin{align*}
   \sin^2 \theta &= \frac{1 - \cos 2\theta}{2} \\
   \cos^2 \theta &= \frac{1 + \cos 2\theta}{2}
   \end{align*}
   \]
(3) Functions

(a) Functions; Vertical Line Test; domain and range of a function; piecewise defined functions; increasing and decreasing functions;

Symmetry: $f(x) = f(-x) \implies f$ is even (i.e symmetric about $y$-axis)

$$\begin{array}{c}
\text{y=f(x)} \\
\text{y} \\
\text{x}
\end{array}$$

$f(-x) = -f(x) \implies f$ is odd (i.e symmetric about the origin, hence graph is the same if rotated $180^\circ$)

$$\begin{array}{c}
\text{y=f(x)} \\
\text{y} \\
\text{x}
\end{array}$$

(b) Types of functions (linear, polynomial, rational, power, algebraic, trig, exponential, logarithmic, transcendental) and how to identify.

(4) Transformations of Functions $y = f(x)$

(I) Vertical Shift $(c > 0)$

(a) $y = f(x) + c \implies \text{shift } f(x) \text{ vertically } c \text{ units up.}$

(b) $y = f(x) - c \implies \text{shift } f(x) \text{ vertically } c \text{ units down.}$

(II) Horizontal Shift $(c > 0)$

(a) $y = f(x - c) \implies \text{shift } f(x) \text{ horizontally } c \text{ units right.}$

(b) $y = f(x + c) \implies \text{shift } f(x) \text{ horizontally } c \text{ units left.}$
(III) **Vertical Stretch/Shrink** \( (c > 1) \)

(a) \( y = cf(x) \) \( \Rightarrow \) *stretch* \( f(x) \) vertically by a factor \( c \). 

(b) \( y = \frac{1}{c} f(x) \) \( \Rightarrow \) *shrink* \( f(x) \) vertically by a factor \( c \).

(IV) **Horizontal Stretch/Shrink** \( (c > 1) \)

(a) \( y = f(cx) \) \( \Rightarrow \) *shrink* \( f(x) \) horizontally by a factor \( c \). 

(b) \( y = f \left( \frac{x}{c} \right) \) \( \Rightarrow \) *stretch* \( f(x) \) horizontally by a factor \( c \).

(V) **Reflections**

(a) \( y = -f(x) \) \( \Rightarrow \) *reflect* \( f(x) \) about \( x\)-axis 

(b) \( y = f(-x) \) \( \Rightarrow \) *reflect* \( f(x) \) about \( y\)-axis
(5) Combinations of functions; composite function \((f \circ g)(x) = f(g(x))\); Law of Exponents; 
\(y = e^x\); exponential functions \(y = a^x\) \((a > 0 \text{ fixed})\):

![Composite function graphs](image1)

(6) One-to-one functions; Horizontal Line Test; inverse functions; finding the inverse of a 1-1 function; graphing inverse functions:

![Inverse function graphs](image2)

(7) Logarithmic functions to base \(a\): \(y = \log_a x\) \((a > 0, a \neq 1)\):

![Logarithmic function graphs](image3)

(8) Logarithm formulas:

\[
\log_a x = y \iff a^y = x \\
\log_a (a^x) = x, \text{ for every } x \in \mathbb{R} \\
a^{\log_a x} = x, \text{ for every } x > 0
\]

(9) Law of Logarithms; Natural logarithm function \(y = \ln x\);

*Change of Base Formula:*

\[
\log_a x = \frac{\ln x}{\ln a}
\]

(10) Slopes of secant lines; approximating slopes of tangent lines using slopes of secant lines; average velocity.
(11) Finite Limits

(a) \( \lim_{x \to a} f(x) = L \)

(b) \( \lim_{x \to a^+} f(x) = L \)

(c) \( \lim_{x \to a^-} f(x) = L \)

Recall: \( \lim_{x \to a} f(x) = L \iff \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L \)

(12) Infinite Limits

(a) \( \lim_{x \to a} f(x) = \infty \)

(b) \( \lim_{x \to a^+} f(x) = \infty \)

(c) \( \lim_{x \to a^-} f(x) = \infty \)

(Similar definitions for \(-\infty\))

Remark: The line \( x = a \) is a vertical asymptote of \( f(x) \) if at least one of \( \lim_{x \to a} f(x) \) or \( \lim_{x \to a^+} f(x) \) or \( \lim_{x \to a^-} f(x) \) is \( \infty \) or \(-\infty\).
(13) Limit Laws: computing limits using Limit Laws;
Squeeze Theorem: If \( f(x) \leq g(x) \leq h(x) \) and \( \lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L \), then \( \lim_{x \to a} g(x) = L \)

(14) \( f \) continuous at \( a \) (i.e. \( \lim_{x \to a} f(x) = f(a) \)); \( f \) continuous on an interval; \( f \) continuous from the left at \( a \) (i.e. \( \lim_{x \to a^-} f(x) = f(a) \)) or continuous from the right at \( a \) (i.e. \( \lim_{x \to a^+} f(x) = f(a) \));
jump discontinuity, removable discontinuity, infinite discontinuity:

\[
\text{y=f}(x)
\]

Limit Composition Theorem: If \( f \) is continuous at \( b \), where \( \lim_{x \to a} g(x) = b \), then
\[
\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right).
\]

(15) Limits at Infinity

(a) \( \lim_{x \to \infty} f(x) = L \)

(b) \( \lim_{x \to -\infty} f(x) = L \)

Remark: The line \( y = L \) is a horizontal asymptote of \( f(x) \).

(16) Intermediate Value Theorem: If \( f \) is continuous on the interval \([a, b]\) and \( N \) is any value between \( f(a) \) and \( f(b) \), then there is a number \( c \), with \( a < c < b \) such that \( f(c) = N \).

(i.e., \( f(x) \) takes on every value between \( f(a) \) and \( f(b) \))