Fall 2011

MA 16100
Study Guide - Exam # 1

(1) Review of Algebra:

(a) Equalities, inequalities, absolute values

(b) Distance between $P(x_1, y_1)$ and $P(x_2, y_2)$ is $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(c) Equations of lines:

 (i) Point-Slope Form: $y - y_1 = m(x - x_1)$

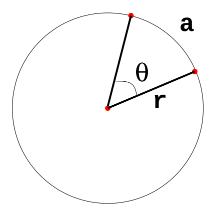
 (ii) Slope-Intercept Form: $y = mx + b$

(d) $L_1 || L_2 \iff m_1 = m_2$; $L_1 \perp L_2 \iff m_2 = -\dfrac{1}{m_1}$

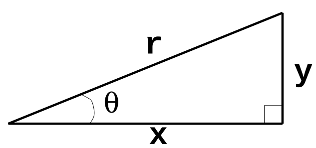
(e) Equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$.

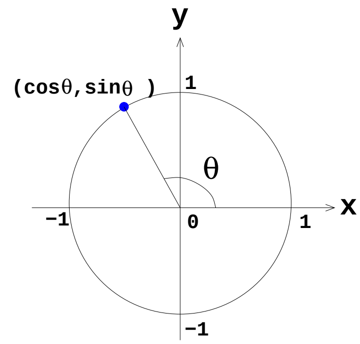
(2) Review of Trigonometry:

(a) Length of arc subtended by θ : $a = r\theta$



(b) Definition of trig functions $(\cos\theta, \sin\theta, \tan\theta, \csc\theta, \sec\theta, \cot\theta)$; basic trig values of well-know angles.





(c) Basic trig identities (including):

(i) $\begin{cases} \sin^2\theta + \cos^2\theta = 1 \\ \tan^2\theta + 1 = \sec^2\theta \end{cases}$

(ii) $\begin{cases} \sin(-\theta) = -\sin\theta \\ \cos(-\theta) = \cos\theta \end{cases}$

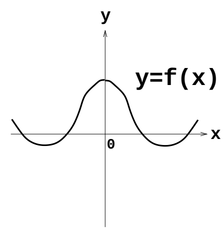
(iii) $\begin{cases} \sin(\theta + 2\pi) = \sin\theta \\ \cos(\theta + 2\pi) = \cos\theta \end{cases}$

(iv) $\begin{cases} \sin 2\theta = 2\sin\theta\,\cos\theta \\ \cos 2\theta = \cos^2\theta - \sin^2\theta \end{cases}$

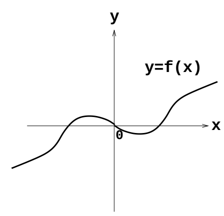
(v) $\begin{cases} \sin^2\theta = \dfrac{1 - \cos 2\theta}{2} \\ \\ \cos^2\theta = \dfrac{1 + \cos 2\theta}{2} \end{cases}$

(3) Functions

(a) Functions; `Vertical Line Test`; domain and range of a function; piecewise defined functions; increasing and decreasing functions;
Symmetry: $f(x) = f(-x) \implies f$ is even (i.e symmetric about y-axis)



$f(-x) = -f(x) \implies f$ is odd (i.e symmetric about the origin, hence graph is the same if rotated 180°)



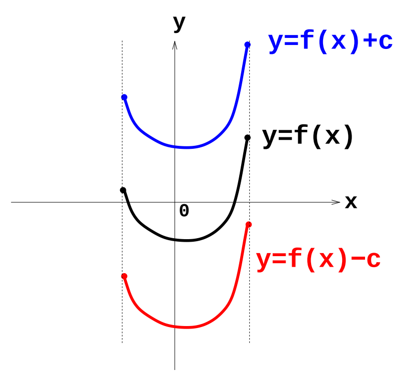
(b) Types of functions (linear, polynomial, rational, power, algebraic, trig, exponential, logarithmic, transcendental) and how to identifty.

(4) Transformations of Functions $y = f(x)$

(I) Vertical Shift $(c > 0)$

(a) $y = f(x) + c \implies$ *shift* $f(x)$ vertically c units *up*.

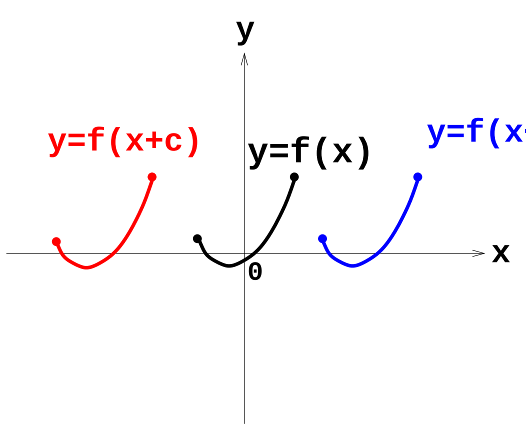
(b) $y = f(x) - c \implies$ *shift* $f(x)$ vertically c units *down*.



(II) Horizontal Shift $(c > 0)$

(a) $y = f(x - c) \implies$ *shift* $f(x)$ horizontally c units *right*.

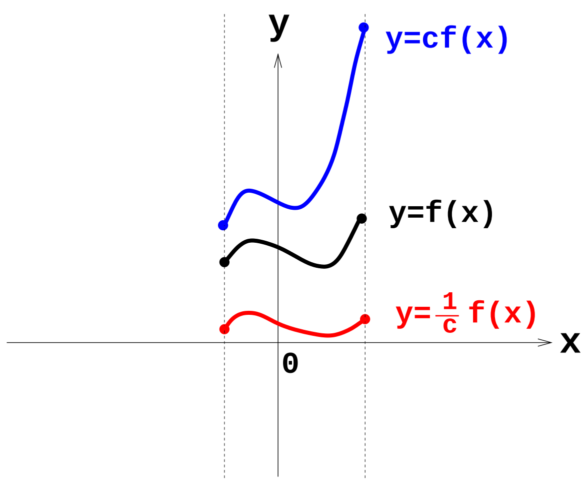
(b) $y = f(x + c) \implies$ *shift* $f(x)$ horizontally c units *left*.



(III) <u>Vertical Stretch/Shrink</u> $(c > 1)$

(a) $y = cf(x) \implies$ *stretch* $f(x)$ vertically by a factor c .

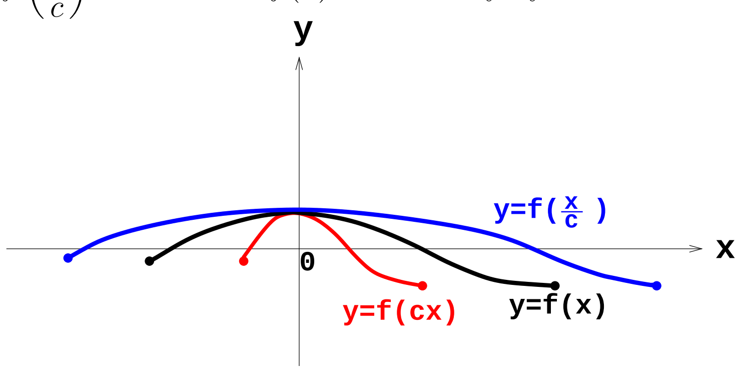
(b) $y = \dfrac{1}{c}\,f(x) \implies$ *shrink* $f(x)$ vertically by a factor c.



(IV) <u>Horizontal Stretch/Shrink</u> $(c > 1)$

(a) $y = f(cx) \implies$ *shrink* $f(x)$ horizontally by a factor c .

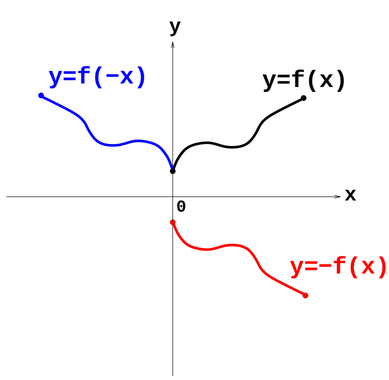
(b) $y = f\left(\dfrac{x}{c}\right) \implies$ *stretch* $f(x)$ horizontally by a factor c .



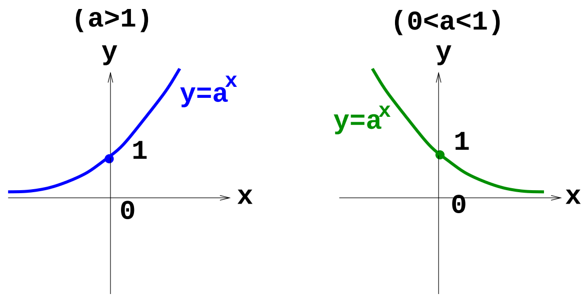
(V) <u>Reflections</u>

(a) $y = -f(x) \implies$ *reflect* $f(x)$ about x-axis

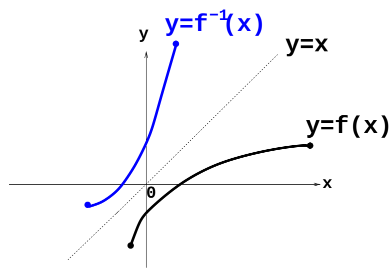
(b) $y = f(-x) \implies$ *reflect* $f(x)$ about y-axis



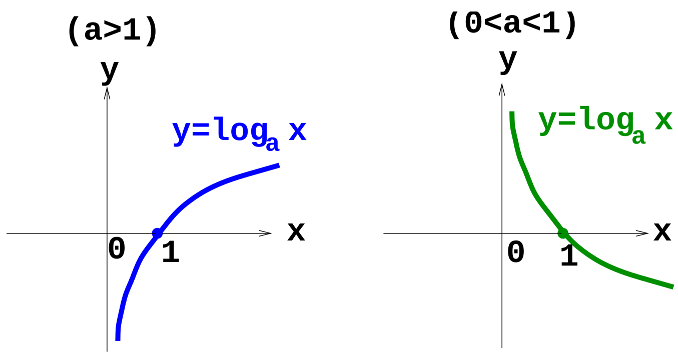
(5) Combinations of functions; composite function $(f \circ g)(x) = f(g(x))$; Law of Exponents; $y = e^x$; exponential functions $y = a^x$ $(a > 0$ fixed):



(6) One-to-one functions; `Horizontal Line Test`; inverse functions; finding the inverse of a 1-1 function; graphing inverse functions:



(7) Logarithmic functions to base a: $y = \log_a x$ $(a > 0,\ a \neq 1)$:



(8) Logarithm formulas:

$$\log_a x = y \iff a^y = x$$
$$\log_a(a^x) = x, \text{ for every } x \in \mathbb{R}$$
$$a^{\log_a x} = x, \text{ for every } x > 0$$

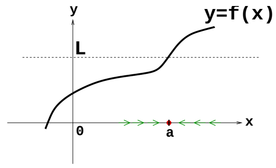
(9) Law of Logarithms; Natural logarithm function $y = \ln x$;

Change of Base Formula: $\boxed{\log_a x = \frac{\ln x}{\ln a}}$

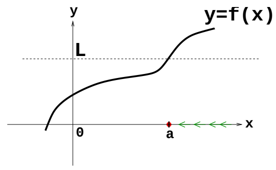
(10) Slopes of secant lines; approximating slopes of tangent lines using slopes of secant lines; average velocity.

(11) <u>Finite Limits</u>

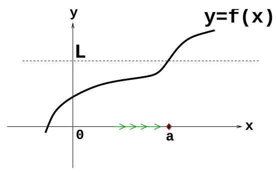
(a) $\lim_{x \to a} f(x) = L$



(b) $\lim_{x \to a^+} f(x) = L$



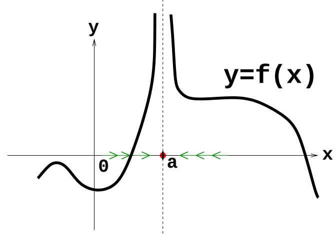
(c) $\lim_{x \to a^-} f(x) = L$



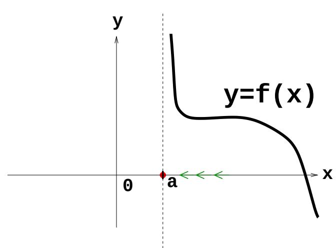
Recall: $\boxed{\lim_{x \to a} f(x) = L \iff \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L}$

(12) <u>Infinite Limits</u>

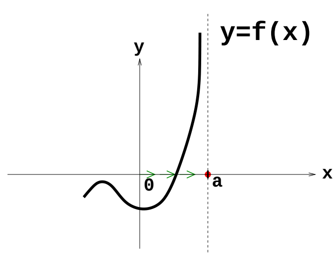
(a) $\lim_{x \to a} f(x) = \infty$



(b) $\lim_{x \to a^+} f(x) = \infty$



(c) $\lim_{x \to a^-} f(x) = \infty$



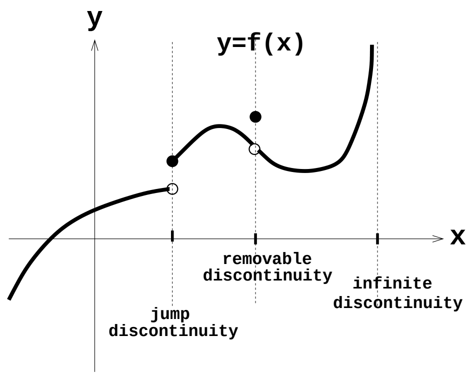
(Similar definitions for $-\infty$)

Remark: The line $x = a$ is a *vertical asymptote* of $f(x)$ if at least one of $\lim_{x \to a} f(x)$ or $\lim_{x \to a^+} f(x)$ or $\lim_{x \to a^-} f(x)$ is ∞ or $-\infty$.

(13) `Limit Laws`; computing limits using Limit Laws;

`Squeeze Theorem` : If $f(x) \leq g(x) \leq h(x)$ and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$

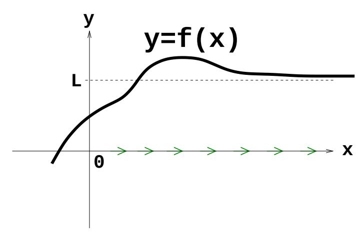
(14) f continuous at a (i.e. $\lim_{x\to a} f(x) = f(a)$); f continuous on an interval; f continuous from the left at a (i.e. $\lim_{x\to a^-} f(x) = f(a)$) or continuous from the right at a (i.e. $\lim_{x\to a^+} f(x) = f(a)$); jump discontinuity, removable discontinuity, infinite discontinuity:



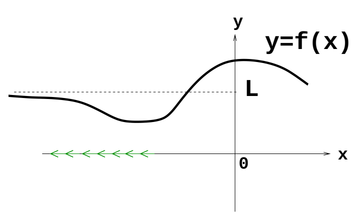
`Limit Composition Theorem`: If f is continuous at b, where $\lim_{x\to a} g(x) = b$, then $\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right)$.

(15) **<u>Limits at Infinity</u>**

(a) $\lim_{x\to\infty} f(x) = L$

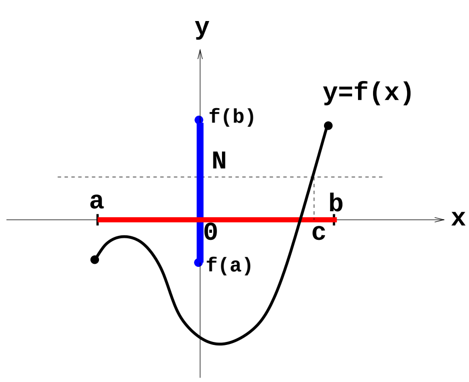


(b) $\lim_{x\to -\infty} f(x) = L$



<u>Remark</u>: The line $y = L$ is a *horizontal asymptote* of $f(x)$.

(16) <u>`Intermediate Value Theorem`</u>: If f is continuous on the interval $[a, b]$ and N is any value between $f(a)$ and $f(b)$, then there is a number c, with $a < c < b$ such that $f(c) = N$.



(i.e., $f(x)$ takes on every value between $f(a)$ and $f(b)$)