Fall 2011

MA 16100
Study Guide - Exam # 3

(1) Method to solve *Related Rates* problems:

1. Read problem carefully several times
2. Draw a picture (if possible) and label.
3. Write down given rate; write down desired rate.
4. Find an equation relating the variables.
5. Use Chain Rule to differentiate equation w.r.t to time and solve for desired rate.

(2) The linear approximation (or tangent line approximation) to a function $f(x)$ at $x = a$ is $L(x) = f(a) + f'(a)(x-a)$; Approximation formula : $\boxed{f(x) \approx f(a) + f'(a)(x-a)}$, for x near a; if $y = f(x)$, the differential of y is $dy = f'(x)\,dx$.

(3) Hyperbolic Trig Functions

(a) Definitions:

(i) $\cosh x = \dfrac{e^x + e^{-x}}{2}$ (ii) $\sinh x = \dfrac{e^x - e^{-x}}{2}$ (iii) $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(b) Derivatives:

(i) $(\cosh x)' = \sinh x$ (ii) $(\sinh x)' = \cosh x$ (iii) $(\tanh x)' = \operatorname{sech}^2 x$

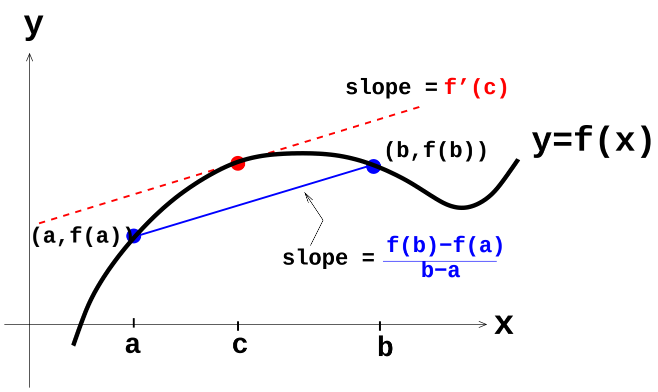
(c) Basic Identites:

(i) $\cosh(-x) = \cosh x$ (ii) $\sinh(-x) = -\sinh x$ (iii) $\cosh^2 x - \sinh^2 x = 1$

(4) Definitions of absolute maximum, absolute minimum, local maxmium, and local minimum; c is a critical number if c is in the domain of f and $f'(c) = 0$ or $'(c)$ DNE; **Extreme Value Theorem**; method for computing absolute extrema for continuous functions over closed intervals.

(5) **Mean Value Theorem**: If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there is a number c, where, $a < c < b$ such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)\,.$$

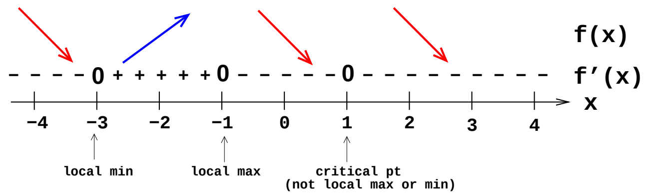


(6) Fact: If $f'(x) = g'(x)$ for all x in I, then $f(x) = g(x) + C$ for all x in I.

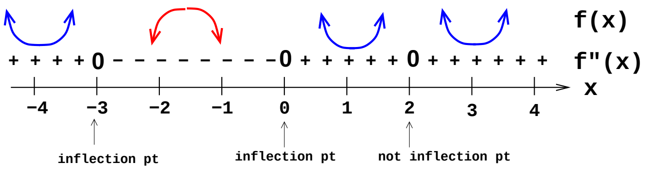
(7) Increasing and decreasing functions, **First Derivative Test**; concave up and concave down; **Concavity Test**; inflection point; **Second Derivative Test**.

(8) Using number line with $f'(x)$ to determine where f is increasing or decreasing, local max and min; using number line with $f''(x)$ to determine where f is concave up or concave down and inflection points. For example:

Increasing/Decreasing



Concave Up/Down



(9) **Indeterminate Forms**:

(a) Indeterminate Form Types: $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0\cdot\infty$, $\infty-\infty$, 0^0, ∞^0, 1^∞

(b) **L'Hopital's Rule**: Let f and g be differentiable and $g(x)\neq 0$ on an open interval I containing a (except possible at a).
If $\lim_{x\to a} f(x)=0$ and $\lim_{x\to a} g(x)=0$ or if $\lim_{x\to a} f(x)=\pm\infty$ and $\lim_{x\to a} g(x)=\pm\infty$, then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)},$$

provided the limit on the right exists or is infinite.

(10) Curve sketching guidelines:

(a) Domain of f
(b) Intercepts (if any)
(c) Symmetry ($f(-x)=f(x)$ for even functions; $f(-x)=-f(x)$ for odd functions)
(d) Asymptotes ($x=a$ is a vertical asymptote if $\lim_{x\to a^+} f(x)$ or $\lim_{x\to a^i} f(x)$ is infinite; $y=L$ is a horizontal asymptote if $\lim_{x\to\infty} f(x)=L$ or $\lim_{x\to-\infty} f(x)=L$)
(e) Intervals where f is increasing and decreasing; local max and local min
(f) Intervals where f is concave up and concave down; ilnflection points

(11) Optimization (Max/Min) Problems Method:

1. Read problem carefully several times.
2. Draw a picture (if possible) and label it.
3. Introduce notation for the quantity, say Q, to be extremized as a function of one or more variables
4. Use information given in problem to express Q as a function of only one variable, say x. Write the domain of Q
5. Use max/min methods to determine the absolute maximum value of Q or the absolute minimum of Q that was asked for in problem.