

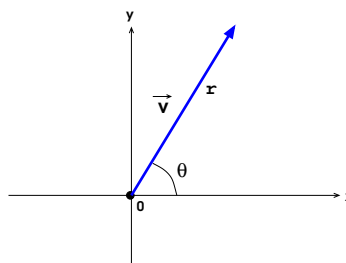
MA 16200

Study Guide - Exam # 1

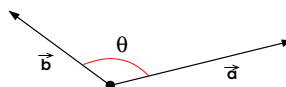
(1) Distance formula $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$; equation of a sphere with center (h, k, l) and radius r : $(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$.

(2) Vectors in \mathbb{R}^2 and \mathbb{R}^3 ; displacement vectors \vec{PQ} ; vector arithmetic; components; Standard basis vectors $\vec{i}, \vec{j}, \vec{k}$, hence $\vec{a} = \langle a_1, a_2, a_3 \rangle = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$; length (magnitude) of a vector $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$; dot (or inner) product of \vec{a} and \vec{b} : $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$; properties of dot products.

(3) Useful Vector: $\vec{v} = (r \cos \theta)\vec{i} + (r \sin \theta)\vec{j}$:



(4) Angle between vectors: $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$:

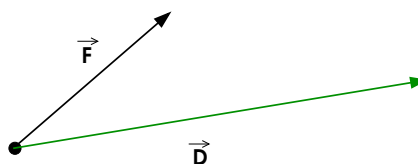
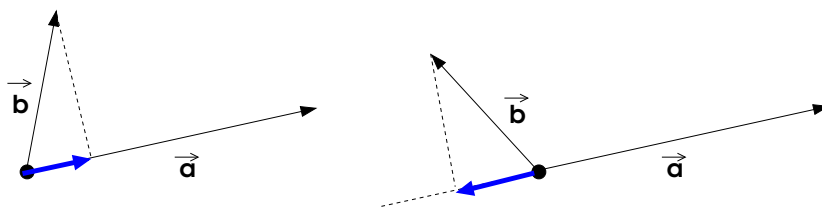


Perpendicular (orthogonal) vectors; direction cosines: $\cos \alpha = \frac{a_1}{|\vec{a}|}$, $\cos \beta = \frac{a_2}{|\vec{a}|}$, $\cos \gamma = \frac{a_3}{|\vec{a}|}$, direction angles α, β, γ .

(5) Vector projection of \vec{b} onto \vec{a} : $\text{proj}_{\vec{a}} \vec{b} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \right) \frac{\vec{a}}{|\vec{a}|}$;

Scalar projection of \vec{b} onto \vec{a} : $\text{comp}_{\vec{a}} \vec{b} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \right)$:

projection of b onto a



Work done by constant force is $W = \vec{F} \cdot \vec{D}$:

(6) Cross product: $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ (defined only for vectors in \mathbb{R}^3); $(\vec{a} \times \vec{b}) \perp \vec{a}$ and

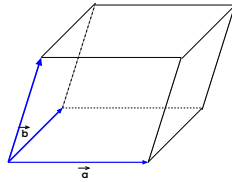
$(\vec{a} \times \vec{b}) \perp \vec{b}$; other properties of cross products; $|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$;

$A = |\vec{a} \times \vec{b}| =$ area of parallelogram spanned by \vec{a} and \vec{b} ;

$A = \frac{1}{2} |\vec{a} \times \vec{b}| =$ area of triangle spanned by \vec{a} and \vec{b} .

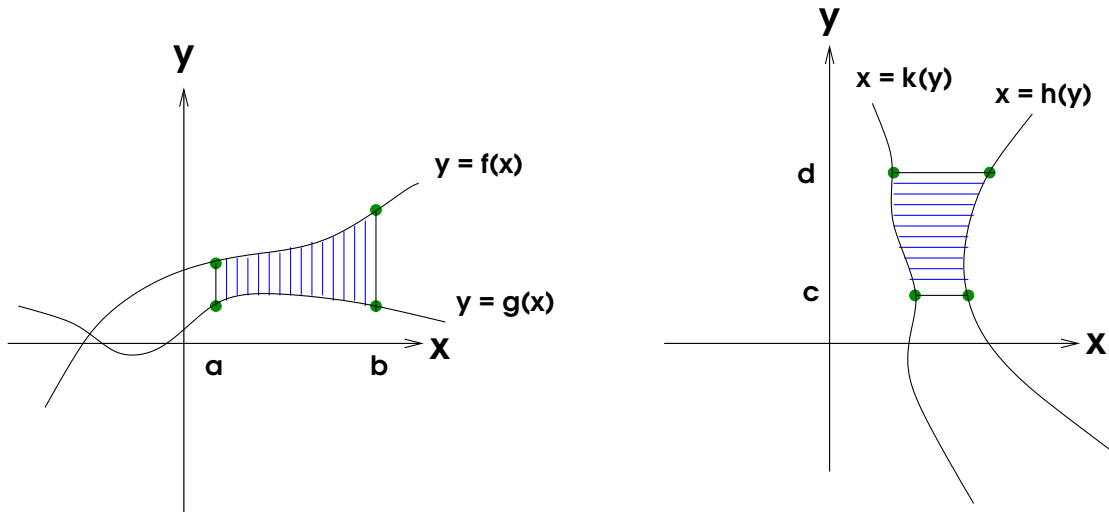


$V = |\vec{a} \cdot (\vec{b} \times \vec{c})| =$ volume of parallelepiped spanned by $\vec{a}, \vec{b}, \vec{c}$:



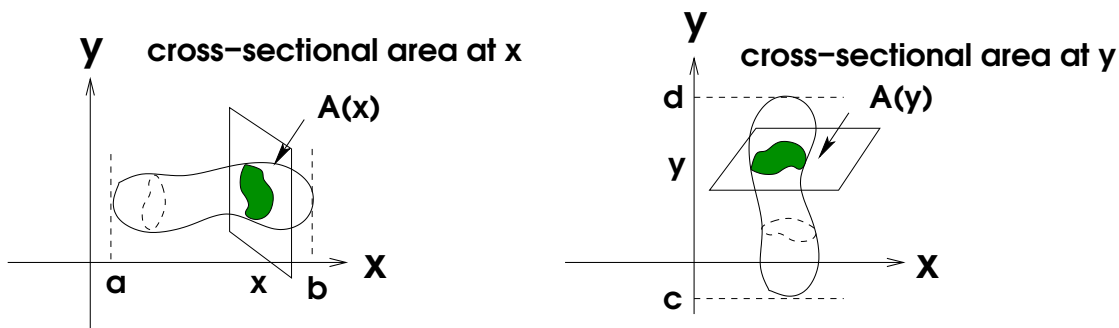
(7) APPLICATIONS OF INTEGRATION

(a) Areas Between Curves: $A = \int_a^b \{f(x) - g(x)\} dx$ or $A = \int_c^d \{h(y) - k(y)\} dy$:

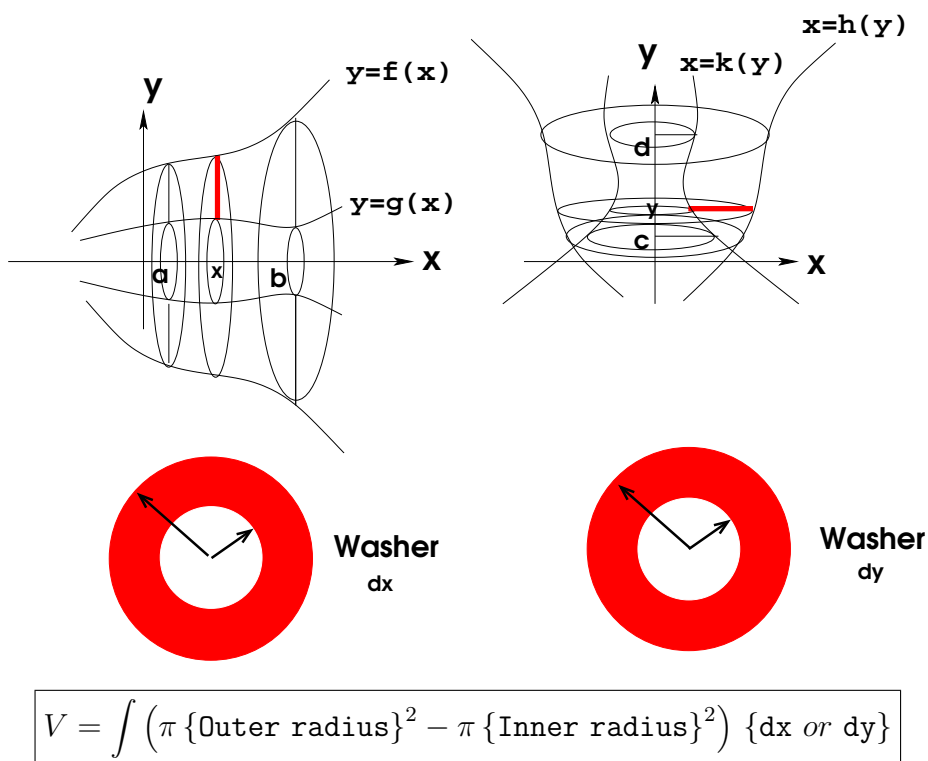
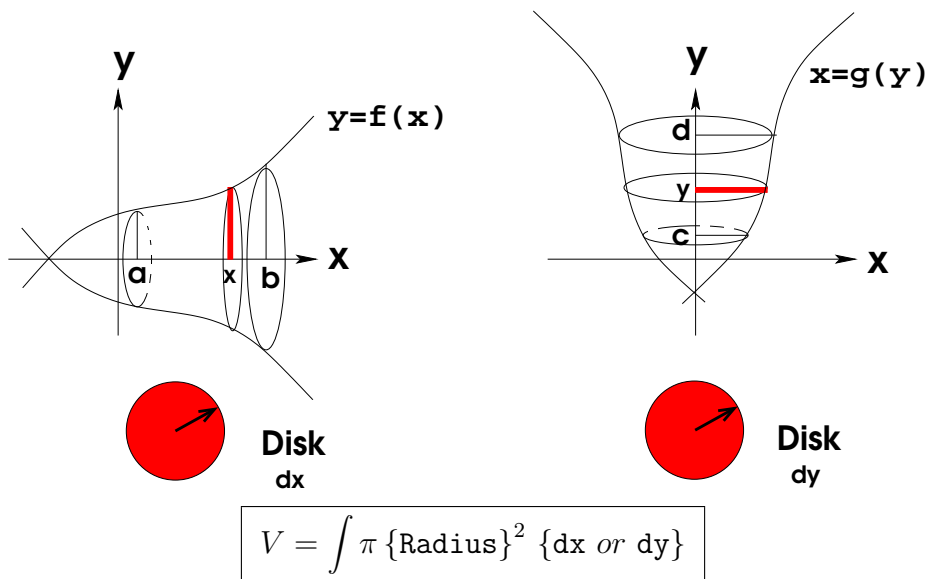


Note: If curves cross, you need to break up into several integrals.

(b) Volumes of Solids by Cross-sectional Areas: $V = \int_a^b A(x) dx$, or $V = \int_c^d A(y) dy$, where
 $A(x) =$ area of the cross-section of the solid with a plane $\perp x$ -axis at the point x , or
 $A(y) =$ area of the cross-section of the solid with a plane $\perp y$ -axis at the point y :



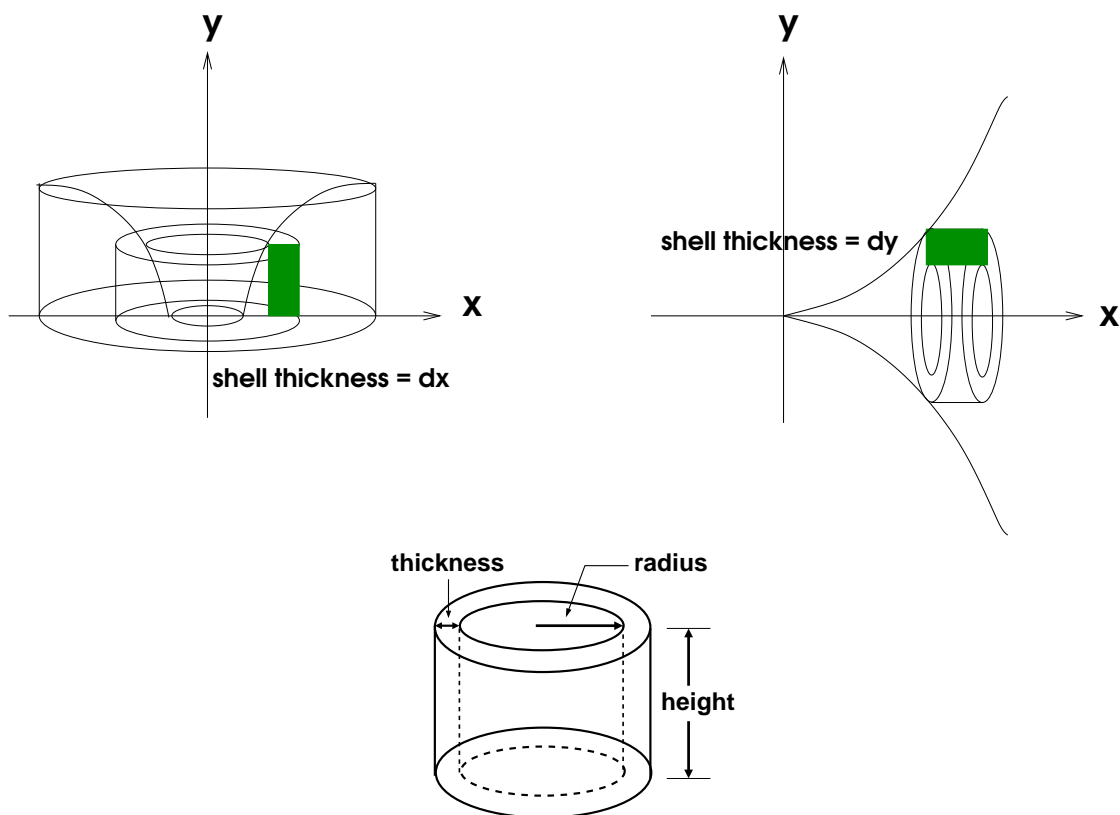
(c) Volumes of Solids of Revolution by DISK METHOD OR WASHER METHOD: Use Disk Method or Washer Method when slices of area are perpendicular to axis of rotation. In either case, the cross-section is always a disk/washer :



IMPORTANT - When to use dx or dy in Disk/Washer method?

- (i) If cross-sections are $\perp x$ - axis, use dx .
- (ii) If cross-sections are $\perp y$ - axis, use dy .

- (d) Volumes of Solids of Revolution by CYLINDRICAL SHELLS METHOD: Use Cylindrical Shells Method when slices of area are parallel to axis of rotation. Shell thickness is always either dx or dy .



$$V = \int_a^b 2\pi \{\text{shell radius}\} \{\text{shell height}\} \{\text{shell thickness}\}$$

- (e) If force F is constant and distance object moved along a line is d , then Work is $W = Fd$. Here are the English and Metric systems compared:

Quantity	English System	Metric System
Mass m	slug (= lb-sec ² /ft)	kilogram kg
Force F	pounds (lbs)	Newtons N (= kg-m/sec ²)
Distance d	feet	meters m
Work W	ft-lbs	Joules J (= kg-m ² /sec ²)
g	32 ft/sec ²	9.8 m/sec ²

If the force is variable, say $f(x)$, then Work $W = \int_a^b f(x) dx$; Hooke's Law: $f_s(x) = kx$; work done compressing/stretching springs, emptying tanks, pulling up chains.

- (f) Average of a function over an interval: $f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$;

Mean Value Thm for Integrals: $f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx = f(c)$, for some $a \leq c \leq b$.

(8) TECHNIQUES OF INTEGRATION

(a) Simple Substitution: $\int_a^b f(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$ (let $u = g(x)$)

(b) Integration by Parts: $\int u dv = uv - \int v du$; the **LIATE** rule:

let $u = \mathbf{L}^{\text{og}} \mathbf{I}^{\text{nv trig}} \mathbf{A}^{\text{lg}} \mathbf{T}^{\text{rig}} \mathbf{E}^{\text{xp}}$.

(c) Trig Integrals: Integrals of the type $\int \sin^m x \cos^n x dx$ and $\int \tan^m x \sec^n x dx$

Some useful trig identities:

(i) $\sin^2 \theta + \cos^2 \theta = 1$

(ii) $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$ and $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$

(iii) $\sin 2\theta = 2 \sin \theta \cos \theta$

(iv) $\tan^2 \theta + 1 = \sec^2 \theta$

Some useful trig integrals:

(i) $\int \tan u du = \ln |\sec u| + C$

(ii) $\int \sec u du = \ln |\sec u + \tan u| + C$

(d) Trig integrals of the form: $\int \sin mx \sin nx dx$, $\int \cos mx \cos nx dx$, $\int \sin mx \cos nx dx$,
use these trig identities:

$$\sin A \sin B = \frac{1}{2} \{ \cos(A - B) - \cos(A + B) \}$$

$$\cos A \cos B = \frac{1}{2} \{ \cos(A - B) + \cos(A + B) \}$$

$$\sin A \cos B = \frac{1}{2} \{ \sin(A - B) + \sin(A + B) \}$$

(e) Trigonometric Substitutions:

<i>Expression*</i>	<i>Trig Substitution</i>	<i>Identity needed</i>
$\sqrt{a^2 - x^2}$	$x = a \sin \theta$	$1 - \sin^2 \theta = \cos^2 \theta$
$\sqrt{a^2 + x^2}$	$x = a \tan \theta$	$1 + \tan^2 \theta = \sec^2 \theta$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta$	$\sec^2 \theta - 1 = \tan^2 \theta$

* Or powers of these expressions.