Fall 2011

MA 16200
Study Guide - Exam # 2

(1) Integration via Partial Fractions: Use for (proper) rational functions $\dfrac{R(x)}{Q(x)}$;
If *degree* $R(x) \geq$ *degree* $Q(x)$, i.e. rational function is improper, then do long division before using partial fractions.

(2) Integration via Clever Substitutions/Using Integral Tables: Use a substitution to transform integral into a form to be able to use another integral techniques (`Substitution Method`, `Integration by Parts`, `Trig Integrals`, `Trig Subsitution Method`, etc) or use integral tables.

(3) Approximating definite integrals $\int_a^b f(x)\,dx$.
Let $\Delta x = \dfrac{b-a}{n}$, $x_k = a + k\,\Delta x$ and $\overline{x}_k = \frac{1}{2}(x_{k-1} + x_k)$ (Note that $x_0 = a$ and $x_n = b$)

(a) Midpoint Rule: $\int_a^b f(x)\,dx \approx M_n = (\Delta x)\,[f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)]$

(b) Trapezoidal Rule: $\int_a^b f(x)\,dx \approx T_n = \left(\dfrac{\Delta x}{2}\right)[f(x_0) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_{n-1}) + f(x_n)]$

(c) Simpson's Rule: Only works for n even.

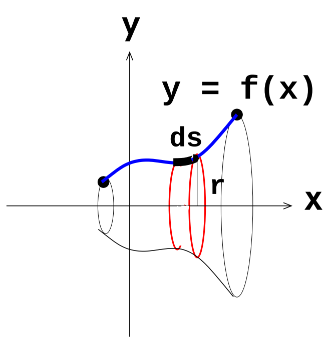
$$\int_a^b f(x)\,dx \approx S_n = \left(\frac{\Delta x}{3}\right)[f(x_0) + 4f(x_2) + 2f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

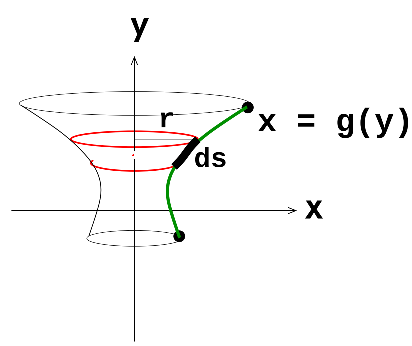
(4) Improper integrals: **Type I** (unbounded intervals) $\int_a^\infty f(x)\,dx$, $\int_{-\infty}^b f(x)\,dx$ or $\int_{-\infty}^\infty f(x)\,dx$;
Improper integrals of **Type II** (discontinuous integrand at one or both endpoints) $\int_a^b f(x)\,dx$.

Comparison Theorem: Let $f(x)$ and $g(x)$ be continuous for $x \geq a$.
(a) If $0 \leq f(x) \leq g(x)$ for $x \geq a$ and $\int_a^\infty g(x)\,dx$ converges $\Longrightarrow \int_a^\infty f(x)\,dx$ also converges.
(b) If $0 \leq g(x) \leq f(x)$ for $x \geq a$ and $\int_a^\infty g(x)\,dx$ diverges $\Longrightarrow \int_a^\infty f(x)\,dx$ also diverges.

(5) Arc length $L = \int_a^b \sqrt{1 + (f'(x))^2}\,dx$ or $L = \int_c^d \sqrt{1 + (g'(y))^2}\,dy$.

(6) Surface area of revolution: $S = \int 2\pi\,\{\texttt{ribbon radius}\}\,ds$ or $S = \int 2\pi r\,ds$,
where $ds = \sqrt{1 + (f'(x))^2}\,dx$ or $ds = \sqrt{1 + (g'(y))^2}\,dy$.





(7) Center of mass of a system of discrete masses $m_1, m_2, \cdots, m_n$ located at $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$ is $(\overline{x}, \overline{y})$, where

$$\overline{x} = \frac{M_y}{M} = \frac{\sum_{k=1}^{n} m_k x_k}{\sum_{k=1}^{n} m_k}, \qquad \overline{y} = \frac{M_x}{M} = \frac{\sum_{k=1}^{n} m_k y_k}{\sum_{k=1}^{n} m_k}$$

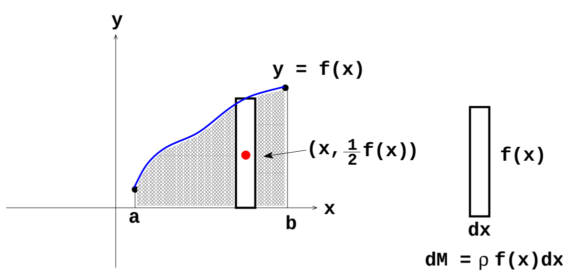
$M_x =$ moment of system about the $x-$axis; $M_y =$ moment of system about the $y-$axis;
$M =$ total mass of the system.

(8) Moments, center of mass (center of mass = *centroid* if density $\rho =$ constant).

(a) Lamina defined by $y = f(x)$, $a \le x \le b$ and $\rho =$ constant:

$$\overline{x} = \frac{M_y}{M} = \frac{\int_a^b x\rho f(x)\,dx}{\int_a^b \rho f(x)\,dx} = \frac{\int_a^b x f(x)\,dx}{\int_a^b f(x)\,dx}$$

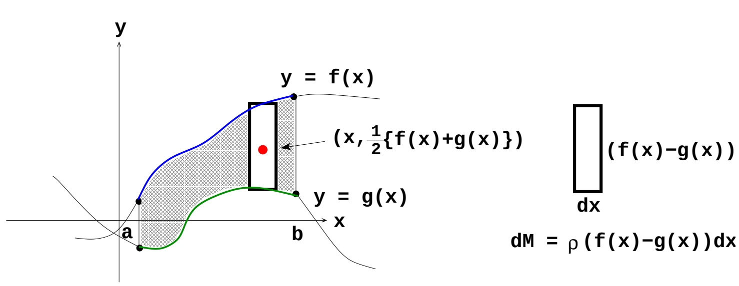
$$\overline{y} = \frac{M_x}{M} = \frac{\int_a^b \frac{1}{2}\rho\,\{f(x)\}^2\,dx}{\int_a^b \rho f(x)\,dx} = \frac{\int_a^b \frac{1}{2}\{f(x)\}^2\,dx}{\int_a^b f(x)\,dx}$$



(b) Lamina between two curves by $y = f(x)$, $y = g(x)$, $a \le x \le b$ and $\rho =$ constant:

$$\overline{x} = \frac{M_y}{M} = \frac{\int_a^b x\rho(f(x) - g(x))\,dx}{\int_a^b \rho(f(x) - g(x))\,dx} = \frac{\int_a^b x(f(x) - g(x))\,dx}{\int_a^b (f(x) - g(x))\,dx}$$

$$\overline{y} = \frac{M_x}{M} = \frac{\int_a^b \frac{1}{2}\rho\left(\{f(x)\}^2 - \{g(x)\}^2\right)\,dx}{\int_a^b \rho(f(x) - g(x))\,dx} = \frac{\int_a^b \frac{1}{2}\left(\{f(x)\}^2 - \{g(x)\}^2\right)\,dx}{\int_a^b (f(x) - g(x))\,dx}$$



(9) Sequences; limits of sequences; Limit Laws for Sequences; monotone sequences (increasing and decreasing); bounded sequences; `Monotone Sequence Theorem`.

(10) Additional useful limit theorems:

(a) *Theorem*: If $\lim_{x\to\infty} f(x) = L$ and $f(n) = a_n$, then $\lim_{n\to\infty} a_n = L$.

(b) *Squeeze Theorem for Sequences*: If $a_n \leq b_n \leq c_n$ for all $n \geq N_0$ with $a_n \longrightarrow L$ and $c_n \longrightarrow L$, then $b_n \longrightarrow L$.

(c) *Theorem*: If $a_n \longrightarrow L$ and f is continuous at L, then $f(a_n) \longrightarrow f(L)$.

(11) Infinite series $\sum_{n=1}^{\infty} a_n$; n^{th} partial sum $s_n = \sum_{k=1}^{n} a_k$; the infinite series $\sum_{n=1}^{\infty} a_n$ **converges** to s if $s_n \to s$; the infinite series **diverges** if $\{s_n\}$ does not have a limit.

(12) `Divergence Test for Series`: If $\lim_{n\to\infty} a_n \neq 0$ or limit fails to exist $\Longrightarrow \sum_{n=1}^{\infty} a_n$ DIVERGES.

(13) Special Infinite Series:

(a) `Harmonic Series`: $\sum_{n=1}^{\infty} \frac{1}{n}$. This series DIVERGES.

(b) `Geometric Series`: $\sum_{n=1}^{\infty} ar^{n-1}$

(i) $\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \cdots = a(1 + r + r^2 + r^3 + \cdots) = \frac{a}{1-r}$, **if** $|r| < 1$.

(ii) $\sum_{n=1}^{\infty} ar^{n-1}$ will DIVERGE **if** $|r| \geq 1$.