(1) Distance formula \( D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \); equation of a sphere with center \((h, k, l)\) and radius \(r\): \((x - h)^2 + (y - k)^2 + (z - l)^2 = r^2\).

(2) Vectors in \(\mathbb{R}^2\) and \(\mathbb{R}^3\); displacement vectors \(\overrightarrow{PQ}\); vector arithmetic; components; Standard basis vectors \(\vec{i}, \vec{j}, \vec{k}\), hence \(\vec{a} = (a_1, a_2, a_3) = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}\); length (magnitude) of a vector \(|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}\); dot (or inner) product of \(\vec{a}\) and \(\vec{b}\): \(\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3\); properties of dot products.

(3) Useful Vector: \(\vec{v} = (r \cos \theta) \vec{i} + (r \sin \theta) \vec{j}\):

(4) Angle between vectors: \(\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}\).

Perpendicular (orthogonal) vectors; direction cosines: \(\cos \alpha = \frac{a_1}{|\vec{a}|}, \cos \beta = \frac{a_2}{|\vec{a}|}, \cos \gamma = \frac{a_3}{|\vec{a}|}\), direction angles \(\alpha, \beta, \gamma\).

(5) Vector projection of \(\vec{b}\) onto \(\vec{a}\): \(\text{proj}_a \vec{b} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}\right) \frac{\vec{a}}{|\vec{a}|}\); Scalar projection of \(\vec{b}\) onto \(\vec{a}\): \(\text{comp}_a \vec{b} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}\right)\).

Work done by constant force is \(W = \vec{F} \cdot \vec{D}\):
(6) Cross product: \( \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \) (defined only for vectors in \( \mathbb{R}^3 \)); \( (\vec{a} \times \vec{b}) \perp \vec{a} \) and \( (\vec{a} \times \vec{b}) \perp \vec{b} \); other properties of cross products; \(|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta \); \( A = |\vec{a} \times \vec{b}| = \) area of parallelogram spanned by \( \vec{a} \) and \( \vec{b} \); \( A = \frac{1}{2}|\vec{a} \times \vec{b}| = \) area of triangle spanned by \( \vec{a} \) and \( \vec{b} \).

\[ V = |\vec{a} \cdot (\vec{b} \times \vec{c})| = \text{volume of parallelopiped spanned by } \vec{a}, \vec{b}, \vec{c}: \]

(7) Applications of Integration

(a) Areas Between Curves: \( A = \int_a^b \{ f(x) - g(x) \} \, dx \) or \( A = \int_c^d \{ h(y) - k(y) \} \, dy \):

(b) Volumes of Solids by Cross-sectional Areas: \( V = \int_a^b A(x) \, dx \), or \( V = \int_c^d A(y) \, dy \), where \( A(x) = \) area of the cross-section of the solid with a plane \( \perp \) \( x \)-axis at the point \( x \), or \( A(y) = \) area of the cross-section of the solid with a plane \( \perp \) \( y \)-axis at the point \( y \):

Note: If curves cross, you need to break up into several integrals.
(c) Volumes of Solids of Revolution by Disk Method or Washer Method: Use Disk Method or Washer Method when slices of area are perpendicular to axis of rotation. In either case, the cross-section is always a disk/washer:

\[ V = \int \pi \{\text{Radius}\}^2 \ {\text{dx or dy}} \]

IMPORTANT - When to use \( dx \) or \( dy \) in Disk/Washer method?

(i) If cross-sections are \( \perp x \) - axis, use \( \frac{dx}{dx} \).
(ii) If cross-sections are \( \perp y \) - axis, use \( \frac{dy}{dy} \).
(d) **Volumes of Solids of Revolution by CYLINDRICAL SHELLS METHOD:** Use Cylindrical Shells Method when slices of area are parallel to axis of rotation. Shell thickness is always either \( dx \) or \( dy \).

\[
V = \int_a^b 2\pi \text{shell radius} \text{shell height} \text{shell thickness}
\]

(e) If force \( F \) is constant and distance object moved along a line is \( d \), then Work is \( W = Fd \). Here are the English and Metric systems compared:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>English System</th>
<th>Metric System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ( m )</td>
<td>slug ((= \text{lb-sec}^2/\text{ft}))</td>
<td>kilogram ( \text{kg} )</td>
</tr>
<tr>
<td>Force ( F )</td>
<td>pounds ((\text{lbs}))</td>
<td>Newtons ( \text{N} ) ((= \text{kg-m/sec}^2))</td>
</tr>
<tr>
<td>Distance ( d )</td>
<td>feet</td>
<td>meters ( \text{m} )</td>
</tr>
<tr>
<td>Work ( W )</td>
<td>ft-lbs</td>
<td>Joules ( \text{J} ) ((= \text{kg-m}^2/\text{sec}^2))</td>
</tr>
<tr>
<td>( g )</td>
<td>32 ft/sec(^2)</td>
<td>9.8 m/sec(^2)</td>
</tr>
</tbody>
</table>

If the force is variable, say \( f(x) \), then Work \( W = \int_a^b f(x) \, dx \); Hooke’s Law: \( f_s(x) = kx \); work done compressing/stretching springs, emptying tanks, pulling up chains.

(f) **Average of a function over an interval:**

\[
f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) \, dx
\]

**Mean Value Thm for Integrals:** \( f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) \, dx = f(c) \), for some \( a \leq c \leq b \).
Techniques of Integration

(a) **Substitution**: \( \int_{a}^{b} f(g(x)) g'(x) \, dx = \int_{g(a)}^{g(b)} f(u) \, du \) (let \( u = g(x) \))

(b) **Integration by Parts**: \( \int u \, dv = uv - \int v \, du \); the LIATE rule:

\[ \text{let } u = \log \text{ Inv trig Alg Trig Exp.} \]

(c) **Trig Integrals**: Integrals of the type \( \int \sin^m x \cos^n x \, dx \) and \( \int \tan^m x \sec^n x \, dx \)

Some useful trig identities:
(i) \( \sin^2 \theta + \cos^2 \theta = 1 \)
(ii) \( \sin^2 \theta = \frac{1 - \cos 2\theta}{2} \) and \( \cos^2 \theta = \frac{1 + \cos 2\theta}{2} \)
(iii) \( \sin 2\theta = 2 \sin \theta \cos \theta \)
(iv) \( \tan^2 \theta + 1 = \sec^2 \theta \)

Some useful trig integrals:
(i) \( \int \tan u \, du = \ln |\sec u| + C \)
(ii) \( \int \sec u \, du = \ln |\sec u + \tan u| + C \)

(d) Trig integrals of the form \( \int \sin mx \sin nx \, dx, \int \cos mx \cos nx \, dx, \int \sin mx \cos nx \, dx, \)

use these trig identities:

\( \sin A \sin B = \frac{1}{2} \{\cos(A - B) - \cos(A + B)\} \)
\( \cos A \cos B = \frac{1}{2} \{\cos(A - B) + \cos(A + B)\} \)
\( \sin A \cos B = \frac{1}{2} \{\sin(A - B) + \sin(A + B)\} \)