MA 16600
Study Guide - Exam # 1

(1) Distance formula $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$; equation of a sphere with center (h, k, l) and radius r: $(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$.

(2) Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$; displacement vectors $\overrightarrow{PQ}$; vector arithmetic; components; Standard basis vectors $\vec{\mathbf{i}}, \vec{\mathbf{j}}, \vec{\mathbf{k}}$; length of a vector $|\vec{\mathbf{a}}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$; dot (or inner) product of $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$: $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_1 b_1 + a_2 b_2 + a_3 b_3$; properties of dot products.

(3) Angle between vectors: $\cos\theta = \dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}|}$; perpendicular vectors; direction cosines: $\cos\alpha = \dfrac{a_1}{|\vec{\mathbf{a}}|}$, $\cos\beta = \dfrac{a_2}{|\vec{\mathbf{a}}|}$, $\cos\gamma = \dfrac{a_3}{|\vec{\mathbf{a}}|}$, direction angles α, β, γ.

(4) Vector projection of $\vec{\mathbf{b}}$ onto $\vec{\mathbf{a}}$: $\text{proj}_{\vec{\mathbf{a}}}\,\vec{\mathbf{b}} = \left(\dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}|}\right)\dfrac{\vec{\mathbf{a}}}{|\vec{\mathbf{a}}|}$; Scalar projection of $\vec{\mathbf{b}}$ onto $\vec{\mathbf{a}}$: $\text{comp}_{\vec{\mathbf{a}}}\,\vec{\mathbf{b}} = \left(\dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}|}\right)$; Work $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{D}}$.

(5) Cross product: $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ (defined **only** for vectors in $\mathbb{R}^3$); $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \perp \vec{\mathbf{a}}$ and $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \perp \vec{\mathbf{b}}$; other properties of cross products; $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = |\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}| \sin\theta$; $A = |\vec{\mathbf{a}} \times \vec{\mathbf{b}}| =$ area of parallelogram spanned by $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$; $A = \frac{1}{2}|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| =$ area of triangle spanned by $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$; $V = |\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})| =$ volume of parallelopiped spanned by $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$.

(6) APPLICATIONS OF INTEGRATION

(a) Areas Between Curves: $A = \int_a^b \{f(x) - g(x)\}\,dx$ or $A = \int_c^d \{h(y) - k(y)\}\,dy$

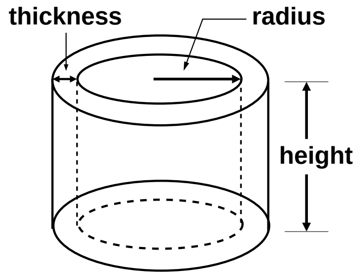
(b) Volumes of Solids by Cross-sectional Areas: $V = \int_a^b A(x)\,dx$, where $A(x) =$ area of the cross-section of the solid with a plane $\perp$ x-axis at the point x;
or $V = \int_c^d A(y)\,dy$, where $A(y) =$ area of the cross-section of the solid with a plane $\perp$ y-axis at the point y

(c) Volumes of Solids of Revolution by "Method of Disks/Washers": $V = \int_a^b A(x)\,dx$, where $A(x) =$ area of the cross-section of the solid with a plane $\perp$ x-axis at the point x;
or $V = \int_c^d A(y)\,dy$, where $A(y) =$ area of the cross-section of the solid with a plane $\perp$ y-axis at the point y.

In either case, the cross-section is always a disk/washer.

Remark #1 - Use Method of Disks/Washers when slices of area are *perpendicular* to axis of rotation.

(d) Volumes of Solids of Revolution by "Method of Cylindrical Shells":



$$V = \int_a^b 2\pi \,\{\texttt{shell radius}\}\,\{\texttt{shell height}\}\,\{\texttt{shell thickness}\}$$

Remark #2 - Use Method of Cyclindrical Shells when slices of area are *parallel* to axis of rotation.

(e) Work $W = \int_a^b f(x)\,dx$; Hooke's Law: $f_{spring}(x) = kx$; work done emptying tanks.

(f) Average of a function over an interval: $f_{ave} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$;

Mean Value Thm for Integrals: $f_{ave} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx = f(c),$ for some $a \le c \le b$.

(7) Techniques of Integration

(a) **Substitution**: $\displaystyle\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$ (let $u = g(x)$)

(b) **Integration by Parts**: $\displaystyle\int u\,dv = uv - \int v\,du$

Choose $u =$ $\mathbf{L}^{\texttt{og}}$ $\mathbf{I}^{\texttt{nv trig}}$ $\mathbf{A}^{\texttt{lg}}$ $\mathbf{T}^{rig}$ $\mathbf{E}^{xp}$