Spring 2009

MA 16600
Study Guide - Exam # 2

(1) Integration Techniques

(a) **Substitution**: $\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du$ (let $u = g(x)$)

(b) **Integration by Parts**: $\int u\, dv = uv - \int v\, du$ (choose $u = \mathbf{L}^{\text{og}}\ \mathbf{I}^{\text{nv trig}}\ \mathbf{A}^{\text{lg}}\ \mathbf{T}^{rig}\ \mathbf{E}^{xp}$)

(c) **Trigonometric Integrals**: $\int \sin^m x \cos^n x\, dx$, $\int \tan^m x \sec^n x\, dx$

(Recall, $\int \sec x\, dx = \ln|\sec x + \tan x| + C$ and $\int \tan x\, dx = \ln|\sec x| + C = -\ln|\cos x| + C$.)

(d) **Trigonometric Substitutions**:

*Expression**	*Trig Substitution*	*Identity*
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

*Or powers of these expressions.

(e) **Partial Fractions**: Use for rational functions $R(x) = \frac{p(x)}{q(x)}$.

If *degree* of $p(x) \geq$ *degree* of $q(x)$, then do long division before using partial fractions.

(f) **Clever Substitutions**: Use a substitution to transform integral into one of the forms (a)-(e) above.

(2) Using Integral Tables.

(3) Approximating definite integrals $\int_a^b f(x)\, dx$.

Let $\Delta x = \frac{b-a}{n}$, $x_i = a + i\,\Delta x$ and $\overline{x}_i = \frac{1}{2}(x_{i-1} + x_i)$

(a) Midpoint Rule: $\int_a^b f(x)\, dx \approx M_n = (\Delta x)\ [f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)]$

(b) Trapezoidal Rule: $\int_a^b f(x)\, dx \approx T_n = \frac{\Delta x}{2}\ [f(x_0) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_{n-1}) + f(x_n)]$

(c) Simpson's Rule: Only works for n even.

$$\int_a^b f(x)\, dx \approx S_n = \frac{\Delta x}{3}\ [f(x_0) + 4f(x_2) + 2f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

(4) Improper integrals: Type I (unbounded intervals) $\int_a^{\infty} f(x)\,dx$, $\int_{-\infty}^{b} f(x)\,dx$ or $\int_{-\infty}^{\infty} f(x)\,dx$; Improper integrals of Type II (discontinuous integrand at one or both endpoints) $\int_a^b f(x)\,dx$.

(5) Arc length $L = \int_a^b \sqrt{1+(f'(x))^2}\,dx$ or $L = \int_c^d \sqrt{1+(g'(y))^2}\,dy$.

(6) Surface area of revolution: $S = \int 2\pi\ \{\texttt{ribbon radius}\}\ ds$ or $S = \int 2\pi r\,ds$,
where $ds = \sqrt{1+(f'(x))^2}\,dx$ or $ds = \sqrt{1+(g'(y))^2}\,dy$.

(7) Center of mass of a system of discrete masses $m_1, m_2, \cdots, m_n$ located at $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$ is $(\overline{x}, \overline{y})$, where

$$\overline{x} = \frac{M_y}{M} = \frac{\sum_{k=1}^{n} m_k x_k}{\sum_{k=1}^{n} m_k}, \qquad \overline{y} = \frac{M_x}{M} = \frac{\sum_{k=1}^{n} m_k y_k}{\sum_{k=1}^{n} m_k}$$

$M_x =$ moment of system about the $x-$axis; $M_y =$ moment of system about the $y-$axis.

(8) Moments and Center of Mass (Centroid) of a plane lamina, with constant density ρ.

(a) Lamina defined by $y = f(x)$, $a \le x \le b$:

$$\overline{x} = \frac{M_y}{M} = \frac{\rho \int_a^b x f(x)\,dx}{\rho \int_a^b f(x)\,dx} = \frac{\int_a^b x f(x)\,dx}{\int_a^b f(x)\,dx}$$

$$\overline{y} = \frac{M_x}{M} = \frac{\rho \int_a^b \frac{1}{2}\{f(x)\}^2\,dx}{\rho \int_a^b f(x)\,dx} = \frac{\int_a^b \frac{1}{2}\{f(x)\}^2\,dx}{\int_a^b f(x)\,dx}$$

(b) Lamina between two curves by $y = f(x)$, $y = g(x)$, $a \le x \le b$:

$$\overline{x} = \frac{M_y}{M} = \frac{\rho \int_a^b x(f(x)-g(x))\,dx}{\rho \int_a^b (f(x)-g(x))\,dx} = \frac{\int_a^b x(f(x)-g(x))\,dx}{\int_a^b (f(x)-g(x))\,dx}$$

$$\overline{y} = \frac{M_x}{M} = \frac{\rho \int_a^b \frac{1}{2}\left(\{f(x)\}^2 - \{g(x)\}^2\right)\,dx}{\rho \int_a^b (f(x)-g(x))\,dx} = \frac{\int_a^b \frac{1}{2}\left(\{f(x)\}^2 - \{g(x)\}^2\right)\,dx}{\int_a^b (f(x)-g(x))\,dx}$$

(9) Sequences; limits of sequences; Limit Laws for Sequences.