(1) Functions \( y = f(x) \), \( x = \text{independent} \) variable, \( y = \text{dependent} \) variable; **domain** of a function (all the values of the independent variable for which the function is defined, i.e., makes sense); **range** of a function (all the values assumed by the dependent variable).

(2) Composition of functions \( f(g(x)) \).

(3) Distance between \( P(x_1, y_1) \) and \( Q(x_2, y_2) \) is \( D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \).

(4) Graph of a function \( y = f(x) \) (plot points); graphing parabolas \( y = Ax^2 + Bx + C \) (parabola opens up if \( A > 0 \), down if \( A < 0 \); vertex is \( x = -\frac{B}{2A} \)); polynomials are functions of the form \( p(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 \); rational functions are quotients of two polynomials \( \frac{p(x)}{q(x)} \); Vertical Line Test (geometric way of determining if a curve is the graph of a function).

(5) **Linear Functions** are functions that change at a constant rate, hence has the form \( y = mx + b \) and its graph is a line; slope \( m \) of the line through \( (x_1, y_1) \) and \( (x_2, y_2) \) is \( m = \frac{y_2 - y_1}{x_2 - x_1} \).

(6) Equations of lines:  
   (i) Slope-Intercept Form: \( y = mx + b \)  
       (Use this one when you know the slope \( m \) and the \( y \)-intercept \( b \).)  
   (ii) Point-Slope Form: \( y - y_0 = m(x - x_0) \)  
       (Use this one when you know the slope \( m \) and a point \( (x_0, y_0) \) on the line.)

(7) Two lines, \( L_1 \) and \( L_2 \), with slopes \( m_1 \) and \( m_2 \), respectively are **parallel** if \( m_1 = m_2 \); they are **perpendicular** if \( m_2 = -\frac{1}{m_1} \).

(8) **Functional Models** (Section 1.4)

   **How to Set Up Model:**  
   (a) READ problem carefully several times to understand what is being asked for and what is given.  
   (b) Draw a diagram (if possible) and label all variables; or identify all variables.  
   (c) Write down the desired function (usually it will have 2 independent variables).  
   (d) Write down information that is given and use this to eliminate one of the variables in Step (c).  
       Now your desired function is a function of **one** independent variable.

(9) Proportionality:  
    - \( Q \) is **directly proportional** to \( x \) if \( Q = kx \), for some constant \( k \)  
    - \( Q \) is **inversely proportional** to \( x \) if \( Q = \frac{k}{x} \), for some constant \( k \)  
    - \( Q \) is **jointly proportional** to \( x \) and \( y \) if \( Q = kxy \), for some constant \( k \)

(10) Law of Supply and Demand

    continue on next page...
(11) **Limits**

(a) \( \lim_{x \to c} f(x) = L \) means that as \( x \to c \) from the right or left of \( c \), the values of \( f(x) \) get closer and closer to \( L \). Note that \( f(x) \) does not have to be defined at the point \( c \).

(b) Know the Algebraic Properties of limits (page 66).

(c) \( \lim_{x \to +\infty} f(x) = L \) means that as \( x \to +\infty \), the values of \( f(x) \) get closer and closer to \( L \) (hence \( y = L \) is a horizontal asymptote).

(d) \( \lim_{x \to -\infty} f(x) = M \) means that as \( x \to -\infty \), the values of \( f(x) \) get closer and closer to \( M \) (hence \( y = M \) is a horizontal asymptote).

(e) To compute limits of the only for the types in (c) and (d), use either the **Dominant Term Rule** or the **High School Rule**:

For example, consider \( \lim_{x \to +\infty} \frac{6x - x^4 - 2}{3x^4 - 1000x^2 + 1} \)

**Dominant Term Rule**: The dominant term in the numerator is \( -x^4 \); while the dominant term in the denominator is \( 3x^4 \). Hence

\[
\lim_{x \to +\infty} \frac{6x - x^4 - 2}{3x^4 - 1000x^2 + 1} = \lim_{x \to +\infty} \frac{-x^4}{3x^4} = \lim_{x \to +\infty} \frac{-1}{3} = -\frac{1}{3}
\]

**High School Rule**: The “highest power” in the denominator is \( x^4 \) so divide numerator and denominator by \( x^4 \) and get:

\[
\lim_{x \to +\infty} \frac{6x - x^4 - 2}{3x^4 - 1000x^2 + 1} = \lim_{x \to +\infty} \frac{6x}{3x^4} - \frac{x^4}{3x^4} - \frac{2}{3x^4} = \lim_{x \to +\infty} \frac{6}{3x^3} - \frac{1}{3} - \frac{2}{x^4} = -\frac{1}{3}
\]

(f) One-sided limits: \( \lim_{x \to c^+} f(x) = L \) and \( \lim_{x \to c^-} f(x) = M \);

\( \lim_{x \to c} f(x) = L \) exists if and only if both the left and right-hand limits exist and are equal.

(g) \( \lim_{x \to c} f(x) = +\infty \) means as \( x \to c \), the values of \( f(x) \) get larger and larger without bound (\( x = c \) is a vertical asymptote).

(h) \( \lim_{x \to c} f(x) = -\infty \) means as \( x \to c \), the values of \( f(x) \) get larger and larger negatively without bound (\( x = c \) is a vertical asymptote).

(12) \( f(x) \) is **continuous** at \( x = c \) if

(i) \( f(c) \) is defined

(ii) \( \lim_{x \to c} f(x) \) exists

(iii) \( \lim_{x \to c} f(x) = f(c) \)

(13) Difference quotient of \( f(x) \) is \( DQ = \frac{f(x + h) - f(x)}{h} \); it represents either the *average rate of change* of \( f(x) \) from \( x \) to \( x + h \) or it is the *slope of the secant line* through the point \((x, f(x))\) and \((x + h, f(x + h))\).

(14) The **derivative** of \( y = f(x) \) is \( f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \). Other notations for the derivative are \( \frac{df}{dx} \) or \( \frac{dy}{dx} \); the derivative represents the instantaneous rate of change of \( f(x) \) at a point or the slope of the tangent line at a point.