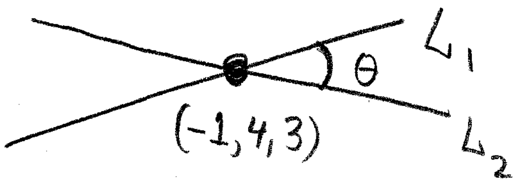
Quiz #1

(10pts) **1** Find a vector $\vec{v}$ perpendicular to the plane containing $(0,0,2)$, $(1,0,2)$ and $(1,2,1)$.

(5pts) **2** The line $L_1: \begin{cases} x = 2-3t \\ y = 4 \\ z = 1+2t \end{cases}$ intersects the line

$L_2: \; x+2 = \dfrac{y-2}{2} = \dfrac{z-1}{2}$ at $(-1,4,3)$. What is the acute angle θ between L_1 and L_2?



A. $\cos^{-1}\left(\frac{4}{3\sqrt{13}}\right)$

B. $\cos^{-1}\left(\frac{1}{3\sqrt{13}}\right)$

C. $\cos^{-1}\left(\frac{-4}{3\sqrt{13}}\right)$

D. $\cos^{-1}\left(\frac{4}{\sqrt{21}}\right)$

E. $\cos^{-1}\left(\frac{7}{3\sqrt{13}}\right)$

(5pts) **3** The plane containing the point $(0,2,-1)$ with normal vector $\vec{v} = \langle 2, 1, 3\rangle$ intersects the y-axis at:

A. $(0,2,0)$

B. $(0,1,0)$

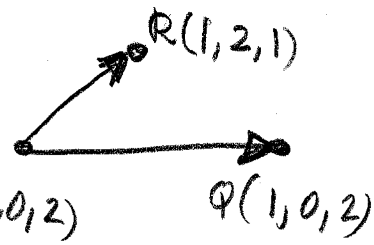
C. $(0,-2,0)$

D. $(0,5,0)$

E. $(0,-1,0)$

Solutions

1



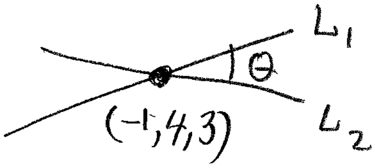
$\vec{a} = \overrightarrow{PQ} = \langle 1, 0, 0 \rangle$

$\vec{b} = \overrightarrow{PR} = \langle 1, 2, -1 \rangle$

$$\vec{v} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 1 & 2 & -1 \end{vmatrix} = \langle (0), -(-1), (2) \rangle$$

$\vec{v} = \langle 0, 1, 2 \rangle$ or any vector $c\langle 0, 1, 2 \rangle$, $c \neq 0$

2



$L_1 \leftarrow$ has direction vector $\vec{n}_1 = \langle -3, 0, 2 \rangle$

$L_2 \leftarrow$ has direction vector $\vec{n}_2 = \langle 1, 2, 2 \rangle$

$$\therefore \cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1|\,|\vec{n}_2|} = \frac{-3+0+4}{\sqrt{13}\ \sqrt{9}} = \frac{1}{3\sqrt{13}} \quad \therefore \theta = \cos^{-1}\left(\frac{1}{3\sqrt{13}}\right)$$

(This angle is acute since $\frac{1}{3\sqrt{13}} > 0$)

3 $P_0(\underset{x_0}{0}, \underset{y_0}{2}, \underset{z_0}{-1})$; $\vec{v} = \langle \underset{a}{2}, \underset{b}{1}, \underset{c}{3} \rangle$

$\therefore$ Equation of plane is $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

so $2(x-0) + 1(y-2) + 3(z-(-1)) = 0$

$2x + (y-2) + 3(z+1) = 0$ This plane intersects the y-axis when $x=0$ and $z=0$ so

$2(0) + y - 2 + 3(0+1) = 0 \Rightarrow y + 1 = 0$ $\quad y = -1$