Quiz #2

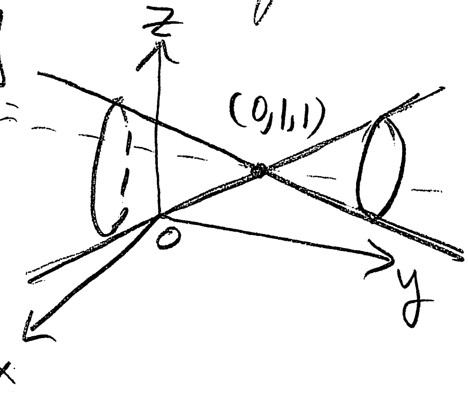
(10 pts) **1** Find parametric equations for the segment from the point $P(1,1,1)$ to $Q(2,-1,3)$. Which of those two points $\left(\frac{1}{3}, \frac{7}{3}, -\frac{1}{3}\right)$ or $\left(\frac{4}{3}, \frac{1}{3}, \frac{5}{3}\right)$ lies on the segment? Why

(5 pts) **2** $\lim\limits_{t \to 1} \left\langle \dfrac{\sin^2(t-1)}{t-1}, \dfrac{2t}{t+1}, \dfrac{\sin \pi t}{\ln t} \right\rangle =$

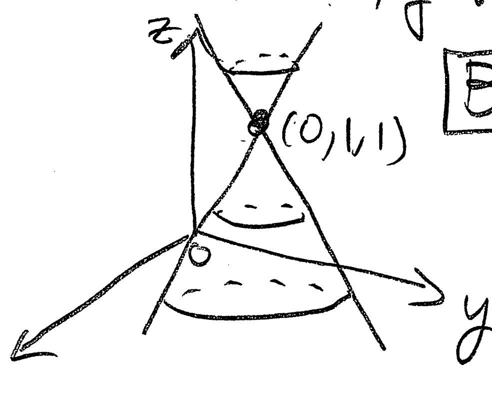
A. $\langle 1, 1, \pi \rangle$
B. $\langle 0, 2, \pi \rangle$
C. $\langle 0, 1, -\pi \rangle$
D. $\langle 1, 1, -\pi \rangle$
E. $\langle 2, 2, -\pi \rangle$

(5 pts) **3** The quadric surface $z^2 - 2z = x^2 + y^2 - 2y$ looks like:

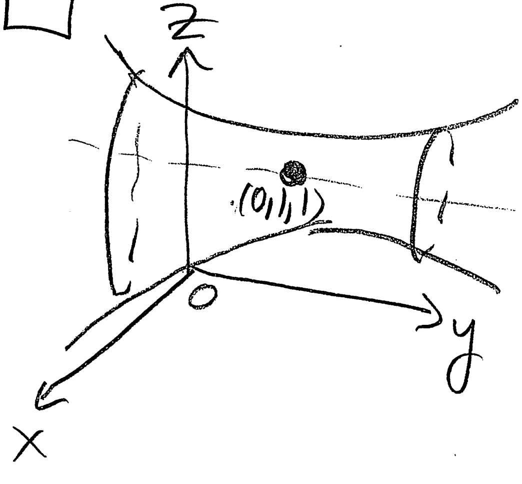
A



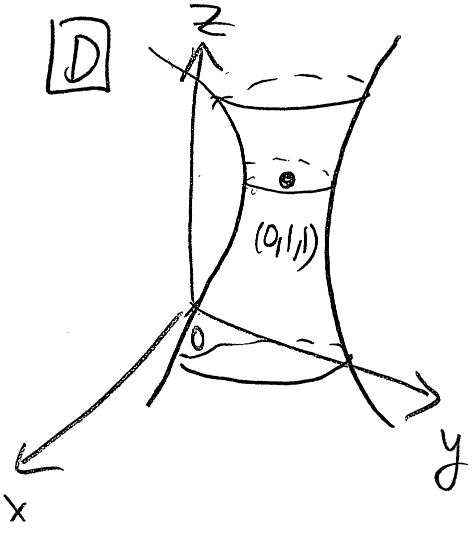
B



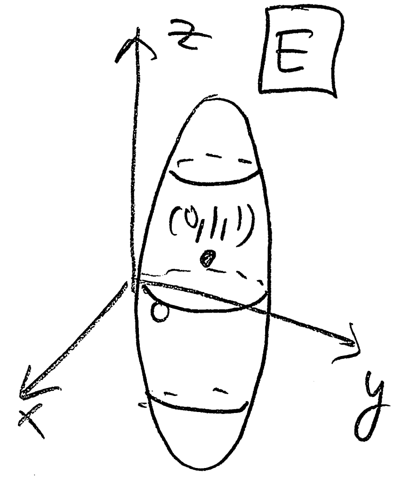
C



D

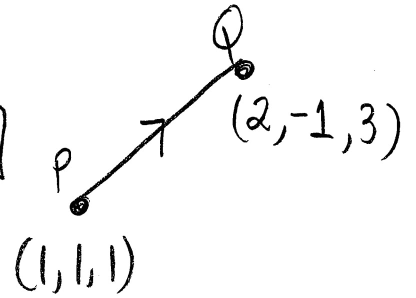


E



Solutions

$\boxed{1}$



$$\Rightarrow \boxed{\vec{r}(t) = t\langle 2,-1,3\rangle + (1-t)\langle 1,1,1\rangle, \quad 0 \le t \le 1}$$

i.e, $\langle x, y, z\rangle = t\langle 2,-1,3\rangle + (1-t)\langle 1,1,1\rangle, \quad 0 \le t \le 1$

$\therefore$ In parametric form: $\begin{cases} x = 1 + t \\ y = 1 - 2t \\ z = 1 + 2t \end{cases} \quad 0 \le t \le 1$

Consider the point $\left(\frac{4}{3}, -\frac{1}{3}, \frac{5}{3}\right) \Rightarrow \begin{cases} \frac{4}{3} = 1 + t \\ y = 1 - 2t \\ z = 1 + 2t \end{cases} \Rightarrow t = \frac{1}{3}$

Since $0 \le \frac{1}{3} \le 1 \Rightarrow \left(\frac{4}{3}, -\frac{1}{3}, \frac{5}{3}\right)$ does lie on the segment

Now consider the point $\left(\frac{1}{3}, \frac{7}{3}, -\frac{1}{3}\right) \Rightarrow \begin{cases} \frac{1}{3} = 1 + t \\ \frac{7}{3} = 1 - 2t \\ -\frac{1}{3} = 1 + 2t \end{cases} \Rightarrow t = -\frac{2}{3}$

$\therefore \left(\frac{1}{3}, \frac{7}{3}, -\frac{1}{3}\right)$ does not lie on the line segment between P and Q (Note: it does lie on the line thru P, Q however)

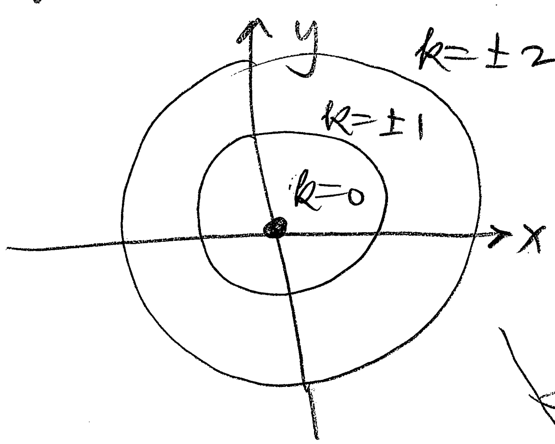
$\boxed{2}$ $\lim\limits_{t\to 1} \left\langle \dfrac{\sin^2(t-1)}{(t-1)}, \dfrac{2t}{t+1}, \dfrac{\sin \pi t}{\ln t} \right\rangle$

$= \lim\limits_{t\to 1} \left\langle \dfrac{2\sin(t-1)\cos(t-1)}{1}, 1, \dfrac{\pi \cos \pi t}{\frac{1}{t}} \right\rangle = \langle 0, 1, -\pi\rangle$

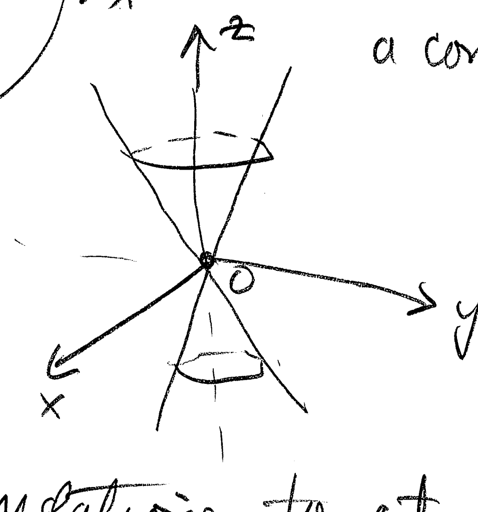
$\boxed{3}$ $z^2 - 2z = x^2 + y^2 - 2y \Rightarrow (z-1)^2 = x^2 + (y-1)^2$ This surface is the surface $z^2 = x^2 + y^2$, except instead of center being at $(0,0,0)$ it is at $(0,1,1)$. So sketch $z^2 = x^2 + y^2$, then translate 1 unit along y axis and 1 unit along z axis:

$z^2 = x^2 + y^2$ is a (double cone):

Look at intersections with planes $z = k$ $(-\infty < k < \infty)$ to get circles:



And, since ~~the intersection~~ with the plane $y=0$ are 2 lines $z^2 = x^2$, the surface is a cone:



Now do the translations to get

$\boxed{B}$