Quiz #3

(10 pts) $\boxed{1}$ Find the length of the curve

$$\vec{r}(t) = (2t)\vec{i} + \left(\tfrac{4}{3}t^{\frac{3}{2}}\right)\vec{j} + \left(\tfrac{1}{2}t^2\right)\vec{k}, \quad 0 \le t \le 2$$

(5 pts) $\boxed{2}$ Let C be the curve given by

$\vec{r}(t) = \langle 4\sqrt{t}, t, (5-t^2) \rangle$ for $t > 0$.

At what point does the tangent line to C at $(4,1,4)$ intersect the xy-plane?

A. $(0,1,0)$

B. $(4\sqrt{5}, \sqrt{5}, 0)$

C. $(2,1,0)$

D. $(8,3,0)$

E. $(0,-1,0)$

(5 pts) $\boxed{3}$ A particle has velocity $\vec{v}(t) = \langle 2, -4t, 3t \rangle$.

At $t=1$, the position vector is $\vec{r}(1) = \langle 3, 1, 5 \rangle$.

What is $\vec{r}(0)$?

A. $\langle 1, 3, 4 \rangle$

B. $\langle 1, 3, \frac{14}{3} \rangle$

C. $\langle 2, -2, 1 \rangle$

D. $\langle 2, 2, -1 \rangle$

E. $\langle 1, 3, \frac{7}{2} \rangle$

Solutions

1 See Exam #1, Fall 2007 #7:

$\vec{r}(t) = \langle 2t, \frac{4}{3}t^{3/2}, \frac{1}{2}t^2 \rangle, \ 0 \le t \le 2$

$\therefore L = \int_0^2 |\vec{r}\,'(t)|\,dt = \int_0^2 |\langle 2, 2t^{1/2}, t\rangle|\,dt$

$= \int_0^2 \sqrt{2^2 + (2t^{1/2})^2 + t^2}\,dt = \int_0^2 \sqrt{4 + 4t + t^2}\,dt$

$= \int_0^2 \sqrt{(2+t)^2}\,dt = \int_0^2 |2+t|\,dt = \int_0^2 (2+t)\,dt$

$= \left(2t + \frac{t^2}{2}\right)\Big|_{t=0}^{2} = \left(4 + \frac{4}{2}\right) = \boxed{6}$

2 See Exam #1, Fall 2011, #1:

$C: \vec{r}(t) = \langle 4\sqrt{t}, t, (5 - t^2)\rangle$

The point $(4,1,4)$ is on the curve when $t = 1$

Thus the tangent line there is given by

$\vec{R}(t) = \vec{r}(1) + t\,\vec{r}\,'(1)$

$= \langle 4,1,4\rangle + t\,\langle 2, 1, -2\rangle$, since $\vec{r}\,'(t) = \langle \frac{2}{\sqrt{t}}, 1, -2t\rangle$

$\therefore \begin{cases} x = 4 + 2t \\ y = 1 + t \\ z = 4 - 2t \end{cases}$ This line crosses xy-plane when $z = 0$

i.e., $t = 2$

Thus $\begin{cases} x = 8 \\ y = 3 \\ z = 0 \end{cases}$

$\boxed{(8,3,0)}$

(cont'd)

3 See Exam #1, Fall 2008 #8:

$\vec{v}(t) = \vec{r}\,'(t) = \langle 2, -4t, 3t \rangle$

$\Rightarrow \quad \vec{r}(t) = \langle 2t, -2t^2, \frac{3}{2}t^2 \rangle + \vec{C}$

Now $\langle 3, 1, 5 \rangle = \vec{r}(1) = \langle 2, -2, \frac{3}{2} \rangle + \vec{C}$

$\Rightarrow \quad \langle 1, 3, \frac{7}{2} \rangle = \vec{C}$

$\therefore \quad \vec{r}(t) = \langle 2t, -2t^2, \frac{3}{2}t^2 \rangle + \langle 1, 3, \frac{7}{2} \rangle$

Hence $\boxed{\vec{r}(0) = \langle 1, 3, \frac{7}{2} \rangle}$