Quiz #4

(10pts) **1** Let $z = f(x,y) = x^2 + 4y^2 + 1$

(a) Sketch some level curves of f.

(b) Using part (a), sketch the surface $z = x^2 + 4y^2 + 1$.

(5pts) **2** If $f(x,y) = y \ln\left(\frac{y^3}{x}\right) + \tan^{-1}(x^3) + xy^2$, $(x, y > 0)$,

then $\frac{\partial^2 f}{\partial y^2} =$

A. $\ln\left(\frac{y^3}{x}\right)\left(\frac{3y^2}{x}\right) + 2xy$

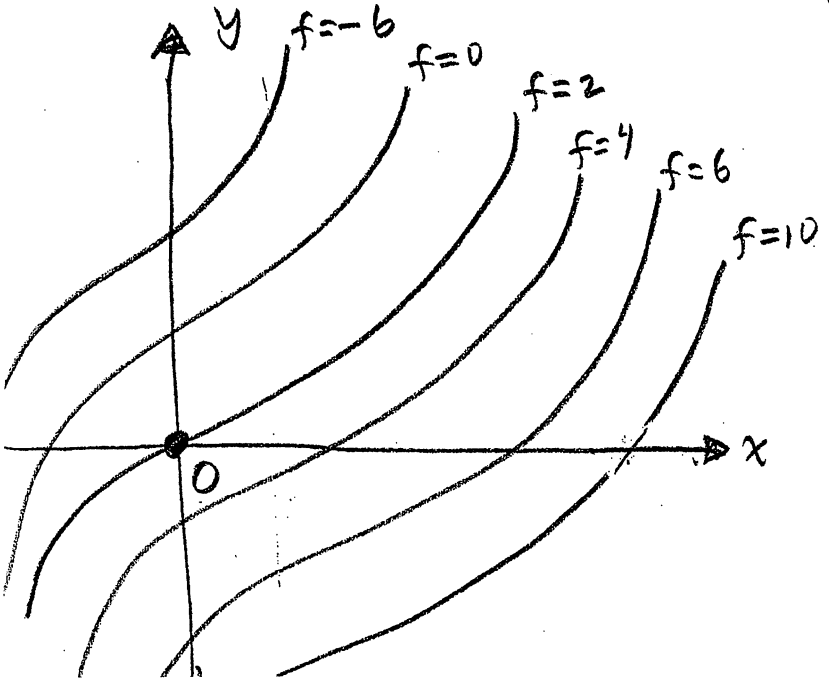
B. $\ln\left(\frac{y^3}{x}\right) + \frac{3x^2}{1+x^6} + 2xy$

C. $3 + 3\ln y + 2x$

D. $3\ln y + 2x$

E. $\frac{3}{y} + 2x$

(5pts) **3** If the level curves of $f(x,y)$ are shown below, what can you conclude about $f_x(0,0)$ and $f_y(0,0)$?



A. $f_x(0,0) > 0$, $f_y(0,0) > 0$

B. $f_x(0,0) < 0$, $f_y(0,0) < 0$

C. $f_x(0,0) > 0$, $f_y(0,0) < 0$

D. $f_x(0,0) < 0$, $f_y(0,0) > 0$

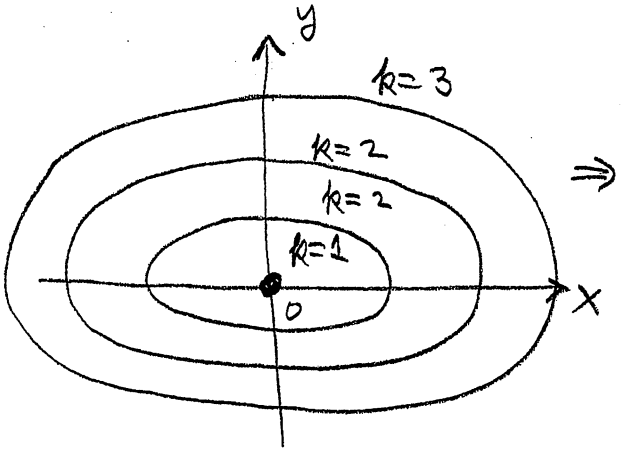
E. $f_x(0,0) = 0$, $f_y(0,0) = 0$

Solutions

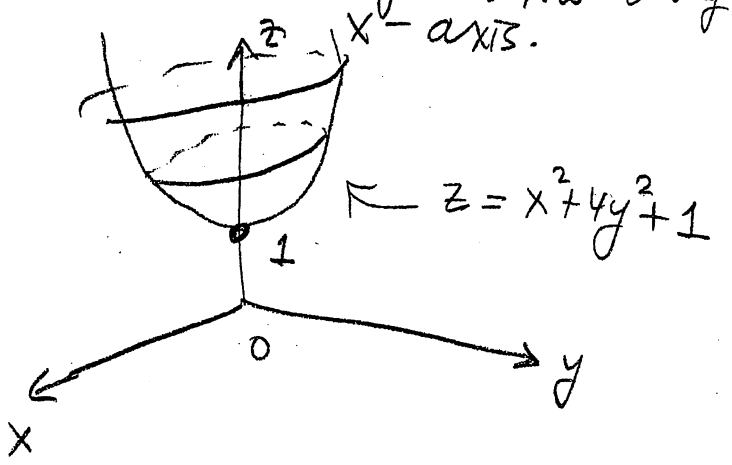
1 $z = f(x,y) = x^2 + 4y^2 + 1$. Level curves $f(x,y) = k$

$\Rightarrow x^2 + 4y^2 + 1 = k \Rightarrow \boxed{x^2 + 4y^2 = (k-1)}$ ← ellipses with center $(0,0)$ major axis along x-axis.

Note $k - 1 \geq 0$ so $k \geq 1$



$\Rightarrow$



2 $f(x,y) = y \ln\left(\frac{y^3}{x}\right) + \tan^{-1}(x^3) + xy^2$

so $f(x,y) = y[3\ln y - \ln x] + \tan^{-1}(x^3) + xy^2$

$\therefore \frac{\partial f}{\partial y} = y\left\{\frac{3}{y}\right\} + \{3\ln y - \ln x\} + 2xy$

$\frac{\partial^2 f}{\partial y^2} = 0 + \frac{3}{y} + 2x = \boxed{\frac{3}{y} + 2x}$

3 To find $f_x(0,0)$, the y-value is fixed: $y = 0$ and along this line f increases

$\therefore \boxed{f_x(0,0) > 0}$

For $f_y(0,0)$, the x-value is fixed: $x = 0$ and along this line f is decreasing

$\therefore \boxed{f_y(0,0) < 0}$