Quiz #5

(10 pts) $\boxed{1}$ If $(0,0,c)$ lies on the tangent plane to the surface $z = x^2 - y^3$ at $(2,1,3)$, find c.

(5 pts) $\boxed{2}$ If $u = (x^2 + x \sin z)$ and $\begin{cases} x = te^{2w} \\ z = t^3 w \end{cases}$

then $\left.\dfrac{\partial u}{\partial w}\right|_{\substack{w=0 \\ t=1}} =$

A. 1

B. 2

C. 3

D. 4

E. 5

(5 pts) $\boxed{3}$ If $w = w(x, s, t)$ is defined implicitly by

$$x^2 + st - \cos w + 8e^{s^2} = w^2 + 1$$

then $\dfrac{\partial w}{\partial t} =$

A. $-\dfrac{(t + 8se^{s^2})}{\sin w - w^2}$

B. $\dfrac{-s}{\sin w - 2w}$

C. $-\dfrac{t}{\sin w - 2w}$

D. $-\dfrac{x^2 + t}{\cos w - w^2}$

E. $\dfrac{16se^{s^2} + t}{\sin w - 2w}$

Solutions

$\boxed{1}$ $z = x^2 - y^3$, $(2, 1, 3)$ — $x_0\ y_0\ z_0$

$z_x(x,y) = 2x$, $z_y(x,y) = -3y^2$

Soln 1 (old way) Eqn of tangent plane is

$$(z - z_0) = z_x(x_0,y_0)(x-2) + z_y(x_0,y_0)(y-1)$$

$$\therefore \boxed{z - 3 = 4(x-2) - 3(y-1)}$$

Hence the pt $(0,0,c)$ lies on this plane when

$$c - 3 = 4(0-2) - 3(0-1) \Rightarrow c = -2$$

Soln 2 (new way with gradients). The surface $z = x^2 - y^3$ is a level surface of $F(x,y,z) = z - x^2 + y^3$ $(F(x,y,z) = 0)$

Hence a normal vector @ $(2,1,3)$ is $\nabla F(2,1,3) = \langle -2x, 3y^2, 1\rangle$ @ $(2,1,3)$

so $\vec{n} = \langle -4, 3, 1\rangle$

$\therefore$ Eqn of tangent plane @ $(2,1,3)$ is

$$\langle x-2, y-1, z-3\rangle \cdot \langle -4, 3, 1\rangle = 0$$

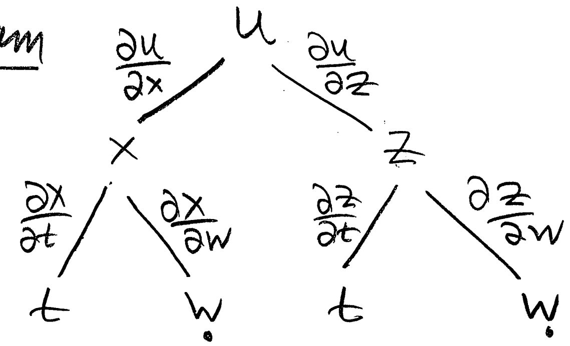
or $-4(x-2) + 3(y-1) + 1(z-3) = 0$, same as above

Now let $x = y = 0$ and $z = c$ $\Rightarrow$ $c = -2$

(cont'd)

$\boxed{2}$ $u = (x^2 + x \sin z)$ and $\begin{cases} x = te^{2w} \\ z = t^3 w \end{cases}$

Tree diagram



when $w=0$, $t=1$
$\Rightarrow x = 1, z = 0$

$$\Rightarrow \frac{\partial u}{\partial w} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial w} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial w} = (2x + \sin z)(2te^{2w}) + (x\cos z)(t^3)$$

$$\therefore \left.\frac{\partial u}{\partial w}\right|_{\substack{w=0\\t=1}} = (2)(2) + (1)(1) = \boxed{5}$$

$\boxed{3}$ $w = w(x,s,t)$ defined implicitly by

$$x^2 + st - \cos w + 8e^{s^2} = w^2 + 1$$

Let $F(x,s,t,w) = x^2 + st - \cos w + 8e^{s^2} - w^2 - 1$

Then $\frac{\partial w}{\partial t} = -\frac{\partial F/\partial t}{\partial F/\partial w} = -\frac{F_t}{F_w} = -\left(\frac{s}{\sin w - 2w}\right)$

or $\frac{s}{2w - \sin w}$