Quiz # 6

(5 pts) **1.** The directional derivative of the function $f(x, y) = 2x + x^2e^{-y}$ at $(1, 0)$ in the direction from $(4, -3)$ to $(7, 1)$ is **A.** $\frac{16}{5}$ **B.** $\frac{8}{5}$ **C.** $\frac{6}{5}$ **D.** $-\frac{3}{5}$ **E.** $\frac{19}{5}$

(5 pts) **2.** In which direction from $(1, 2, 1)$ is $f(x, y, z) = x^2 + 3y - z^2$ increasing the fastest?

A. $< 20, 30, -20 >$ **B.** $< 20, 30, -10 >$ **C.** $< 20, 30, 40 >$

D. $< 20, 40, 20 >$ **E.** $< -2, 3, -1 >$

(10 pts) **3.** Given that $(1, 1)$ and $(-1, -2)$ are critical points of $f(x, y)$ and given the table shown below, which statement about f is true?

(x, y)	f_{xx}	f_{xy}	f_{yx}	f_{yy}
$(1, 1)$	2	4	4	6
$(-1, -2)$	-2	4	4	-12

A. Local max at $(1, 1)$; Saddle point at $(-1, -2)$

B. Local min at $(1, 1)$; Local max at $(-1, -2)$

C. Local max at $(1, 1)$; Local min at $(-1, -2)$

D. Saddle point at $(1, 1)$; Local min at $(-1, -2)$

E. Saddle point at $(1, 1)$; Local max at $(-1, -2)$

Solutions

1 $\vec{v} = \langle 7,1\rangle - \langle 4,-3\rangle = \langle 3,4\rangle$

$\vec{u} = \dfrac{\vec{v}}{|\vec{v}|} = \dfrac{\langle 3,4\rangle}{5}$, $\nabla f = \langle 2 + 2xe^{-y}, -x^2e^{-y}\rangle$

$D_{\vec{u}} f(1,0) = \nabla f(1,0)\cdot\vec{u} = \langle 4,-1\rangle \cdot \langle \frac{3}{5}, \frac{4}{5}\rangle = \boxed{\frac{8}{5}}$

2 f increases fastest in direction of ∇f.

i.e. $\nabla f(1,2,1) = \langle 2x, 3, -2z\rangle \Big|_{(1,2,1)} = \langle 2, 3, -2\rangle$

$\therefore$ the direction is $k\langle 2,3,-2\rangle$ for any $k>0$

hence $\boxed{\langle 20, 30, -20\rangle}$

3 $D(1,1) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}_{(1,1)} = \begin{vmatrix} 2 & 4 \\ 4 & 6 \end{vmatrix} = 12 - 16 = -4 < 0$

$\Rightarrow$ $(1,1)$ is a <u>saddle point</u> ✓

$D(-1,-2) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}_{(-1,-2)} = \begin{vmatrix} -2 & 4 \\ 4 & -12 \end{vmatrix} = 24 - 16 = 8 > 0$

and since $f_{xx}(-1,-2) = -2 < 0$

$\Rightarrow$ f has <u>local max</u> @ $(-1,-2)$ ✓