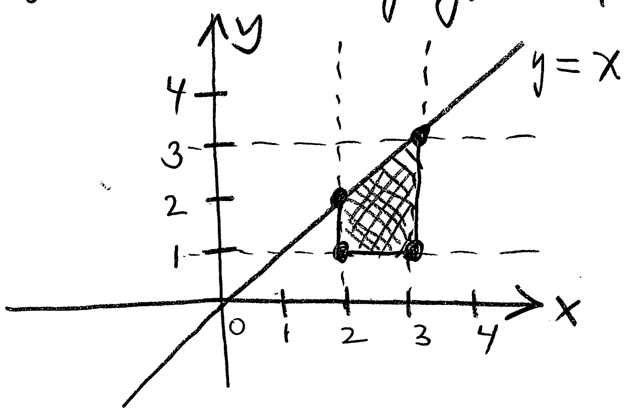
Quiz #7

(10 pts) $\boxed{1}$ Using the <u>Lagrange Multiplier Method</u>, find the max and min values of $f(x,y) = \left(x^3 + \frac{y^3}{3} + 1\right)$ subject to the constraint $x^3 + y^2 = 4$.

(5 pts) $\boxed{2}$ If D is the trapezoidal region below, then $\iint_D 6y\, dA =$



A. 12

B. 14

C. 16

D. 18

E. 20

(5 pts) $\boxed{3}$ $\displaystyle\int_0^4 \int_{\frac{x}{2}}^{2} e^{-y^2}\, dy\, dx =$

A. $1 - e^{-4}$

B. $e^{-4} - 4$

C. $e^{-2} + 1$

D. $e^{-2} - 4$

E. e^{-4}

Solutions

1 Maximize $f(x,y) = x^3 + \frac{1}{3}y^3 + 1$

s.t. $\underbrace{x^3 + y^2}_{g(x,y)} = 4$

Use Lagrange Multiplies

$\Rightarrow \nabla f = \lambda \nabla g \Rightarrow \langle 3x^2, y^2 \rangle = \lambda \langle 3x^2, 2y \rangle$

$$\therefore \begin{cases} 3x^2 = 3\lambda x^2 & (1) \\ y^2 = \lambda(2y) & (2) \\ x^3 + y^2 = 4 & (3) \end{cases}$$

(1) $\Rightarrow 3x^2(1-\lambda) = 0$

$x = 0$ or $\lambda = 1$

$x=0$: $\Downarrow$ by (3) $y = \pm 2$

$\lambda = 1$: By (2): $y^2 = 2y$

i.e. $y(y-2) = 0$

$y = 0$ or $y = 2$

$y=0$: $\therefore$ (3) $\Rightarrow x = 4^{1/3}$

$y=2$: $\therefore$ (3) $\Rightarrow x = 0$

(x,y)	$f(x,y) = x^3 + \frac{1}{3}y^3 + 1$
$(0, 2)$	$\frac{8}{3} + 1 = 11/3$
$(0, -2)$	$-\frac{8}{3} + 1 = -5/3$ ← min value of f on $x^3+y^2=4$
$(4^{1/3}, 0)$	5 ← max value of f on $x^3+y^2=4$

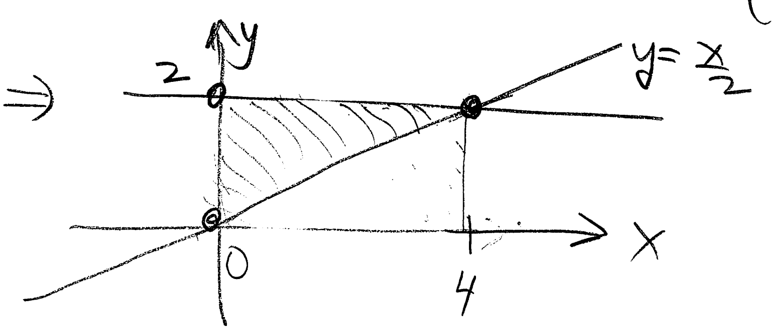
[2] D is Type I region: $\begin{cases} 1 \le y \le x \\ 2 \le x \le 3 \end{cases}$

$$\therefore \iint_D 6y\,dA = \int_2^3 \left(\int_1^x 6y\,dy \right) dx = \int_2^3 3y^2 \Big|_{y=1}^{x} dx = \int_2^4 (3x^2 - 3)\,dx$$

$$= (x^3 - 3x) \Big|_{x=2}^{3} = (3^3 - 9) - (8 - 6) = \boxed{16}$$

[3] $I = \int_0^4 \int_{\frac{x}{2}}^{2} e^{-y^2}\,dy\,dx \Rightarrow D: \begin{cases} \frac{x}{2} \le y \le 2 \\ 0 \le x \le 4 \end{cases}$ Type I

$\Rightarrow$



D is both Type I and II, so change order of integration:

$\therefore$ D: $\begin{cases} 0 \le x \le 2y \\ 0 \le y \le 2 \end{cases}$ Type II

So $$I = \int_0^2 \left(\int_0^{2y} e^{-y^2}\,dx \right) dy = \int_0^2 2y\,e^{-y^2}\,dy = -e^{-y^2} \Big|_{y=0}^{2}$$

$$= (-e^{-4}) - (-1) = \boxed{1 - e^{-4}}$$