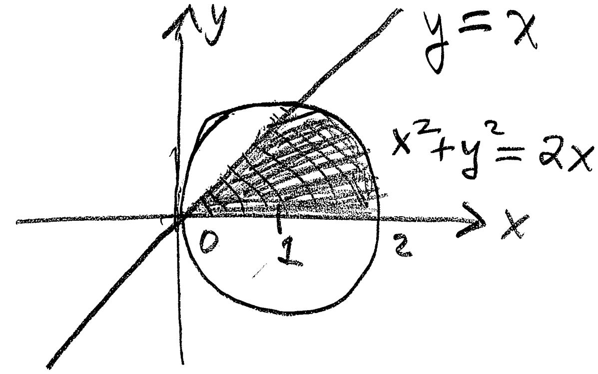
Quiz #8

(10pts) $\boxed{1}$ Find the area of the region D:



(5pts) $\boxed{2}$ The area of that part of the paraboloid $z = x^2 + y^2$ below the plane $z = 4$ is

A. $\frac{\pi}{3}(5^{3/2} - 1)$

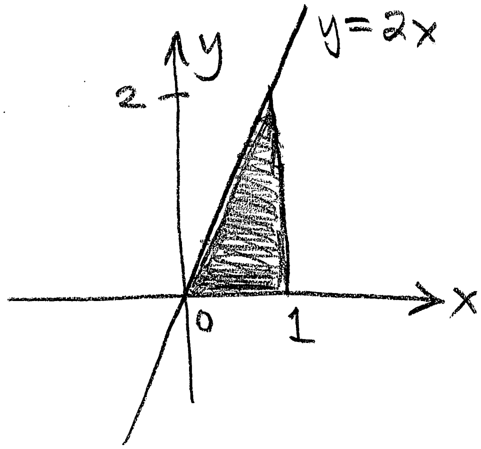
B. $\frac{\pi}{6}(37^{3/2} - 1)$

C. $\frac{\pi}{6}(5^{3/2} - 1)$

D. $\frac{\pi}{6}(17^{3/2} - 1)$

E. $\frac{\pi}{6}$

(5pts) $\boxed{3}$ If the density $\rho(x,y) = 4y$ and the ~~total~~ mass of D shown below is $m = \frac{8}{3}$, then $\bar{x} = ?$



A. $1/3$

B. $2/3$

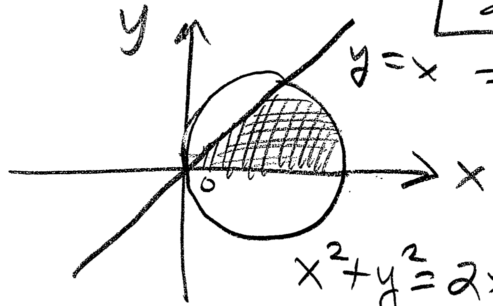
C. $1/2$

D. $3/4$

E. 1

Solutions

1



$y = x \Rightarrow r\sin\theta = r\cos\theta \Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

$x^2 + y^2 = 2x \Rightarrow r^2 = 2r\cos\theta \Rightarrow r = 2\cos\theta$

$\therefore$ In Polar Coordinates $D: \begin{cases} 0 \le r \le 2\cos\theta \\ 0 \le \theta \le \frac{\pi}{4}. \end{cases}$

$$\therefore A = \iint_D dA = \int_0^{\frac{\pi}{4}} \int_0^{2\cos\theta} r\,dr\,d\theta$$

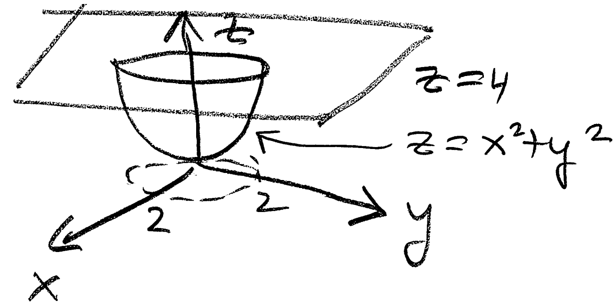
don't forget

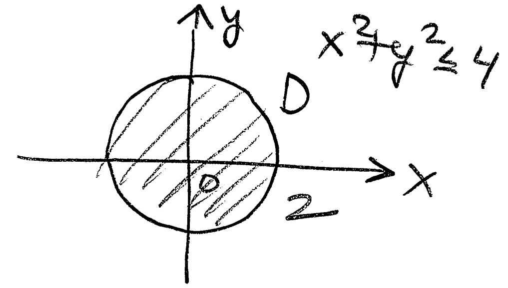
$$= \int_0^{\frac{\pi}{4}} \frac{r^2}{2}\Bigg|_{r=0}^{2\cos\theta} d\theta = \int_0^{\frac{\pi}{4}} \left[\frac{4\cos^2\theta}{2}\right] d\theta = \int_0^{\frac{\pi}{4}} 2\cos^2\theta\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} (1 + \cos 2\theta)\, d\theta = \left(\theta + \frac{1}{2}\sin 2\theta\right)\Bigg|_{\theta=0}^{\frac{\pi}{4}}$$

$$= \underline{\underline{\left(\frac{\pi}{4} + \frac{1}{2}\right)}}$$

$\boxed{2}$





$$\therefore A(S) = \iint_D \sqrt{z_x^2 + z_y^2 + 1}\, dA = \iint_D \sqrt{(2x)^2 + (2y)^2 + 1}\, dA$$

$$= \iint_D \sqrt{4(x^2+y^2)+1}\, dA$$

Use polar coordinates

$$D: \begin{cases} 0 \le r \le 2 \\ 0 \le \theta \le 2\pi \end{cases}$$

$$= \int_0^{2\pi} \int_0^{2} \sqrt{4r^2+1}\; r\,dr\,d\theta$$

$\uparrow$ don't forget

$$= \int_0^{2\pi} \frac{1}{12}(4r^2+1)^{3/2}\Big|_{r=0}^{2} d\theta = \int_0^{2\pi} \frac{1}{12}\left[17^{3/2} - 1\right] d\theta$$

$$= \frac{\pi}{6}\left[17^{3/2} - 1\right]$$