Quiz # 9

(10 pts) **1.** Let E be the solid in the 1^{st} octant between the paraboloid $z = 1 + x^2 + y^2$ and the plane $z = 10$. Set up but do not evaluate the triple integral $\iiint_E (3y + z^2)\, dV$ in *Cylindrical Coordinates*.

(5 pts) **2.** $\displaystyle\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \int_{0}^{8} e^{(x^2+y^2)}\, dz\, dy\, dx =$

A. $2\pi\,(e-1)$ **B.** 4π **C.** $\pi\,(e-1)$ **D.** $4\pi\,(e-1)$ **E.** $10\pi\,(e^2-1)$

(5 pts) **3.** $\displaystyle\int_{0}^{2} \int_{0}^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{0} (4z+1)\, dz\, dy\, dx$ in *Spherical Coordinates* is

A. $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^2 (4\rho\cos\phi + 1)\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

B. $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^{\pi} \int_0^2 (4\rho\cos\phi + 1)\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

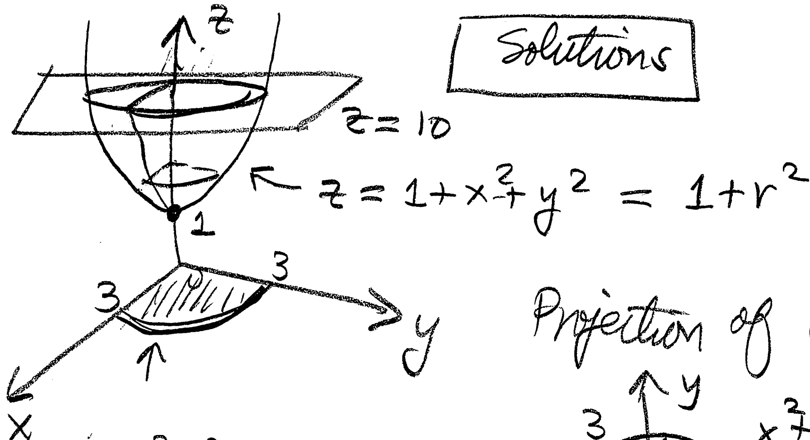
C. $\displaystyle\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \int_0^2 (4\rho\cos\phi + 1)\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

D. $\displaystyle\int_0^{\pi} \int_{\frac{\pi}{2}}^{\pi} \int_0^2 (4\rho\cos\phi + 1)\, \rho \sin\phi\, d\rho\, d\phi\, d\theta$

E. $\displaystyle\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \int_0^2 (4\rho\cos\phi + 1)\, d\rho\, d\phi\, d\theta$

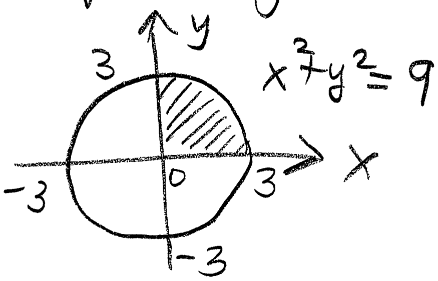
Solutions

1



$$1 + x^2 + y^2 = 10$$
$$x^2 + y^2 = 9$$

Projection of E onto xy plane is



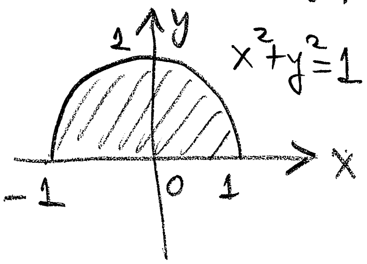
$\therefore$ In CC, $E : \begin{cases} 1 + r^2 \le z \le 10 \\ 0 \le r \le 3 \\ 0 \le \theta \le \frac{\pi}{2} \end{cases}$

Hence $\displaystyle\iiint_E (3y + z^2)\, dV = \int_0^{\frac{\pi}{2}} \int_0^3 \int_{1+r^2}^{10} (3r\sin\theta + z^2)\, r\, dz\, dr\, d\theta$ ✓

2 $\displaystyle\int_{-1}^{1} \int_0^{\sqrt{1-x^2}} \int_0^8 e^{(x^2+y^2)}\, dz\, dy\, dx$

$E : \begin{cases} 0 \le z \le 8 \\ 0 \le y \le \sqrt{1-x^2} \\ -1 \le x \le 1 \end{cases}$

Projection of E onto xy plane is



Thus in CC:

$E : \begin{cases} 0 \le z \le 8 \\ 0 \le r \le 1 \\ 0 \le \theta \le \pi \end{cases}$

(cont'd)

2 (cont'd). Thus,

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{8} e^{(x^2+y^2)}\,dz\,dy\,dx = \int_{0}^{\pi}\int_{0}^{1}\int_{0}^{8} e^{r^2}\,\underset{\uparrow}{r}\,dz\,dr\,d\theta$$

$$= \int_{0}^{\pi}\int_{0}^{1} 8re^{r^2}\,dr\,d\theta$$

$$= \int_{0}^{\pi} 4e^{r^2}\Big|_{r=0}^{1}\,d\theta$$

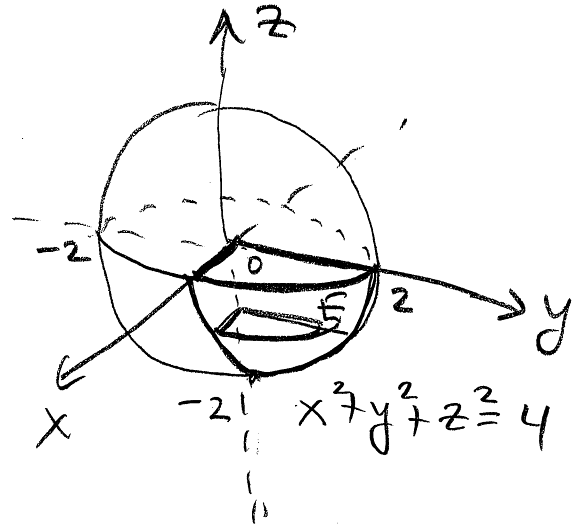
$$= \int_{0}^{\pi} (4e-4)\,d\theta$$

$$= (4e-4)\pi$$

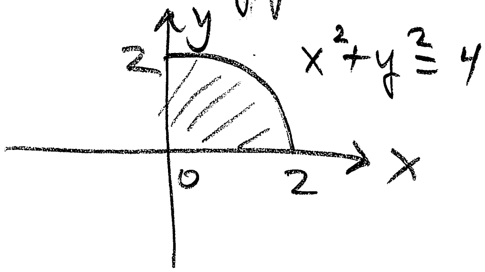
$$= 4\pi(e-1) \checkmark$$

$\boxed{3}$ $$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{0} (4z+1)\, dz\, dy\, dx = I$$

$$\Rightarrow E: \begin{cases} -\sqrt{4-x^2-y^2} \le z \le 0 \\ 0 \le y \le \sqrt{4-x^2} \\ 0 \le x \le 2 \end{cases}$$



Projection in xy plane



$$\therefore E: \begin{cases} 0 \le \rho \le 2 \\ \frac{\pi}{2} \le \phi \le \pi \\ 0 \le \theta \le \frac{\pi}{2} \end{cases}$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \int_0^2 (4\rho\cos\phi + 1)\rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$ ✓