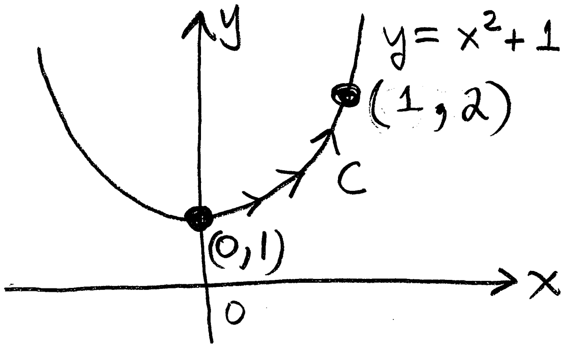
Quiz # 10

(5 pts) **1.** If C is the line segment from $(0,1)$ to $(3,5)$, then $\int_C 3x^2\,ds =$

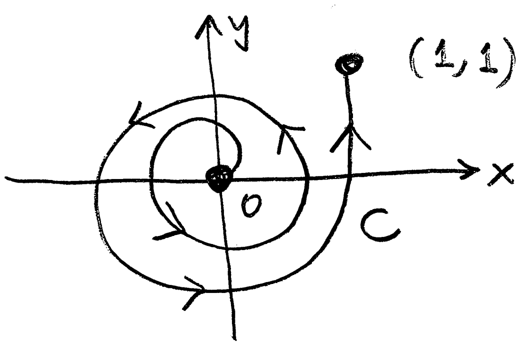
A. 16 **B.** 25 **C.** 45 **D.** 65 **E.** 75

(5 pts) **2.** If C is the curve below, compute $\int_C 10x^2y\,dx - 5x\,dy$.



A. 2 **B.** $\frac{11}{3}$ **C.** 1 **D.** 4 **E.** $\frac{7}{3}$

(10 pts) **3.** Let $\tilde{\mathbf{F}}(x,y) = (2xy^3 - 6x)\,\mathbf{i} + (3x^2y^2 + 4)\,\mathbf{j}$. Given that $\tilde{\mathbf{F}}(x,y) = \nabla f(x,y)$, find a potential function $f(x,y)$ and use the *Fundamental Theorem of Calculus for Line Intergrals* to compute $\int_C \tilde{\mathbf{F}} \cdot d\tilde{\mathbf{r}}$, where C is as shown below.



A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** -1

$\boxed{\text{Solutions}}$

$\boxed{1}$ $C: \vec{r}(t) = t\langle 3,5\rangle + (1-t)\langle 0,1\rangle, \quad 0 \le t \le 1$

$\vec{r}(t) = \langle 3t, 1+4t\rangle$

Or, equivalently, $\vec{r}(t) = \vec{r}_0 + t(\vec{r}_1 - \vec{r}_0) = \langle 0,1\rangle + t\langle 3,4\rangle$

Thus, $C: \begin{cases} x = 3t \\ y = 1+4t \end{cases} \quad (0 \le t \le 1)$

Next, $ds = \sqrt{(x')^2 + (y')^2}\,dt = \sqrt{9+16}\,dt = 5dt$

$\therefore \int_C 3x^2\,ds = \int_0^1 3(3t)^2\{5dt\} = (27)(5)\int_0^1 t^2\,dt$

$= (27)(5)\left(\frac{1}{3}\right) = \boxed{45}$

$\boxed{2}$ $C: \begin{cases} x = t \\ y = 1+t^2 \end{cases}, \quad 0 \le t \le 1$

$\therefore \int_C 10x^2y\,dx - 5x\,dy = \int_0^1 10(t)^2(1+t^2)\{dt\} - 5(t)\{2t\,dt\}$

$= \int_0^1 10t^4\,dt = 2t^5\Big|_0^1 = \boxed{2}$

3 $\vec{F}(x,y) = \nabla f(x,y)$

$\Rightarrow \langle 2xy^3 - 6x,\ 3x^2y^2 + 4 \rangle = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$

Find $f(x,y)$:

Method 1:

$$\begin{cases} \frac{\partial f}{\partial x} = 2xy^3 - 6x \xrightarrow{I_x} f(x,y) = x^2y^3 - 3x^2 + g(y) \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \downarrow D_y \\ \frac{\partial f}{\partial y} = 3x^2y^2 + 4 \quad = \quad \frac{\partial f}{\partial y} = 3x^2y^2 + g'(y) \end{cases}$$

$\therefore\ g'(y) = 4 \Rightarrow g(y) = 4y + C$

Hence $f(x,y) = x^2y^3 - 3x^2 + 4y + C$. All we need is one potential function so let $C = 0$.

Thus $f(x,y) = x^2y^3 - 3x^2 + 4y$ ✓

Method 2:

$$\begin{cases} \frac{\partial f}{\partial x} = 2xy^3 - 6x \xrightarrow{I_x} f(x,y) = x^2y^3 - 3x^2 + g(y) \\ \frac{\partial f}{\partial y} = 3x^2y^2 + 4 \xrightarrow{I_y} f(x,y) = x^2y^3 + 4y + h(x) \end{cases}$$

comparing 2 forms

$\Rightarrow f(x,y) = x^2y^3 - 3x^2 + 4y$ ✓ Same as above

Hence, $\int_C \vec{F} \cdot d\vec{r} = \int_C \nabla f \cdot d\vec{r} \underset{\uparrow \text{FTC}}{=} f(1,1) - f(0,0) = \boxed{2}$