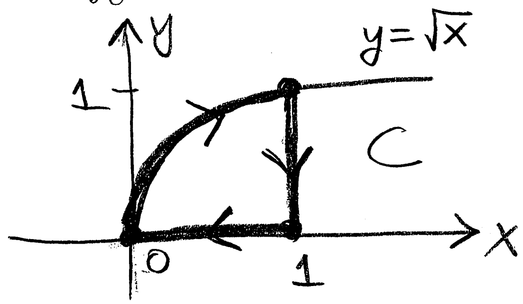
Quiz # 11

(5 pts) **1.** If C is the circle $x^2 + y^2 = 4$ traversed once in a positive sense, use Green's Theorem to evaluate $\int_C -y^3\,dx + x^3\,dy$.

A. 8π **B.** 16π **C.** 4π **D.** -4π **E.** 24π

(10 pts) **2.** If C is the simple closed curve traversed as shown below, compute
$\int_C (y^3 + \sin x)\,dx + (3xy^2 - e^{\pi y} + 15x^2)\,dy$.



A. -8 **B.** -3π **C.** 4 **D.** -12 **E.** 0

(5 pts) **3.** Let $f(x,y,z) = x^2e^{3z} + 2y + 40$ and $\vec{\mathbf{G}} = 2xy\mathbf{i} + (x + z^2)\mathbf{k}$.
If $\alpha = \nabla \times \vec{\mathbf{G}}$ and $\beta = \operatorname{div}(\nabla f)$, then

A. $\alpha = \mathbf{i} + (1 - 2x)\,\mathbf{k};\ \beta = e^{3z}(2 + 9x^2)$ **B.** $\alpha = -\mathbf{j} - 2x\,\mathbf{k};\ \beta = e^{3z}(2 + 9x^2)$

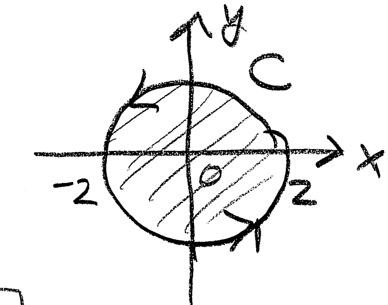
C. $\alpha = -\mathbf{j} - 2x\,\mathbf{k};\ \beta = e^{3z}(2x + 6x^2)$ **D.** $\alpha = < 0, 1, -2x >;\ \beta = e^{3z}(2x + 6x^2)$

E. $\alpha = \mathbf{j} - 2x\,\mathbf{k};\ \beta$ is not defined

Solutions

1

$$\int_C -y^3\,dx + x^3\,dy = \iint_D \left(3x^2 - (-3y^2)\right) dA$$

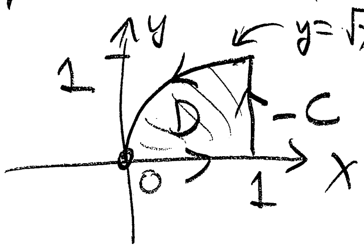


$$= \iint_D 3(x^2+y^2)\,dA = \int_0^{2\pi}\int_0^{2} 3r^2\, r\,dr\,d\theta = \boxed{24\pi}$$

2

$$\int_{-C} (y^3 + \sin x)\,dx + (3xy^2 - e^{\pi y} + 15x^2)\,dy = \iint_D 30x\,dA$$

Need positive direction for Green's Thm



$$= \int_0^1\int_0^{\sqrt{x}} 30x\,dy\,dx = 12$$

$$\therefore \int_C (y^3 + \sin x)\,dx + (3xy^2 - e^{\pi y} + 15x^2)\,dx = \boxed{-12}$$

3 $f(x,y,z) = x^2e^{3z} + 2y + 40$, $\vec{G}(x,y,z) = \langle 2xy, 0, (x+z^2)\rangle$

$$\alpha = \nabla \times \vec{G} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & 0 & (x+z^2) \end{vmatrix} = \langle (\ 0\ ), -(\ 1\ ), (-2x) \rangle$$

$$\therefore \boxed{\alpha = \langle 0, -1, -2x \rangle}$$

$$\beta = \operatorname{div}(\nabla f) = \operatorname{div}\left(\langle 2xe^{3z}, 2, 3x^2e^{3z}\rangle\right) = 2e^{3z} + 0 + 9x^2e^{3z}$$

$$\boxed{\beta = e^{3z}(2 + 9x^2)}$$