Quiz # 12

(10 pts) **1.** The tangent plane to the surface S given by $\tilde{\mathbf{r}}(u, v) = \langle u^2, 3uv, 2u + v\rangle$ at the point $(1, 0, -2)$ intersects the y–axis at

A. $y = -9$ **B.** $y = -3$ **C.** $y = 9$ **D.** $y = 1$ **E.** $y = 0$

(10 pts) **2.** If S is that part of $z = 4 - x^2$ between $y = 0$ and $y = 1$ in the first octant, compute $\iint_S 24x\, dS$.

A. $\frac{1}{2}(6^{\frac{3}{2}} - 1)$ **B.** $2(17^{\frac{3}{2}} - 1)$ **C.** $\frac{1}{2}(5^{\frac{3}{2}} - 1)$ **D.** 48 **E.** $2(5^{\frac{3}{2}} - 1)$

Solutions

1 $S: \vec{r}(u,v) = \langle u^2, 3uv, 2u+v \rangle$

$$\vec{n} = \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2u & 3v & 2 \\ 0 & 3u & 1 \end{vmatrix} = \langle (3v-6u), -(2u), (6u^2) \rangle$$

$$\vec{n} = \langle 3v-6u, -2u, 6u^2 \rangle$$

The point $(1,0,-2)$ satisfies $\begin{cases} x = u^2 = 1 \\ y = 3uv = 0 \\ z = 2u+v = -2 \end{cases} \Rightarrow \begin{matrix} u = -1 \\ v = 0 \end{matrix}$

$\therefore$ Normal vector @ $(u_0, v_0) = (-1, 0)$ is $\vec{n} = \langle 6, 2, 6 \rangle$ ✓

Tangent plane @ $(1, 0, -2)$ is then

$$\langle x-1, y-0, z-(-2) \rangle \cdot \langle 6, 2, 6 \rangle = 0$$

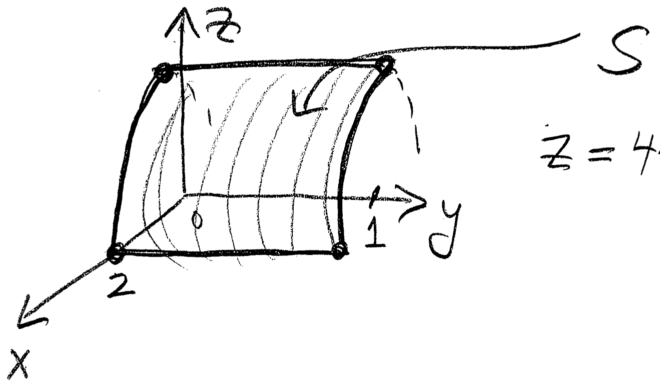
$$\Rightarrow \boxed{6(x-1) + 2y + 6(z+2) = 0}$$

Crosses y-axis when $x = 0$, $z = 0$

so $-6 + 2y + 6(2) = 0$

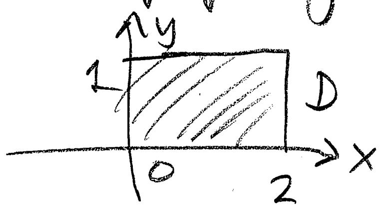
$$\Rightarrow \boxed{y = -3}$$

2



$z = 4 - x^2$

Soln 1: Since S is graph of a function $z = 4 - x^2$ where $(x, y) \in D$:



$$\Rightarrow dS = \sqrt{f_x^2 + f_y^2 + 1}\, dA = \sqrt{(-2x)^2 + 0^2 + 1}\, dA = \sqrt{4x^2+1}\, dA$$

$$\therefore \iint_S 24x \, dS = \iint_D 24x \sqrt{4x^2+1}\, dA$$

$$= \int_0^1 \int_0^2 24x\sqrt{4x^2+1}\, dx\, dy = \int_0^1 2(4x^2+1)^{3/2} \Big|_{x=0}^{2} dy$$

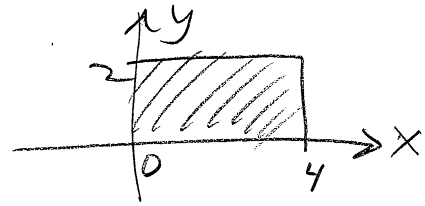
$$= \int_0^1 2(17^{3/2} - 1)\, dy = \boxed{2(17^{3/2} - 1)}$$

(cont'd)

Soln 2: Parameterize S "naturally" by

$$S: \begin{cases} x = x \\ y = y \\ z = 4 - x^2 \end{cases} \quad \text{i.e., } \vec{r}(x,y) = \langle x, y, 4-x^2 \rangle \text{ where } (x,y) \in D$$

and D:



$$dS = |\vec{r}_x \times \vec{r}_y| \, dA = |\langle 1, 0, -2x \rangle \times \langle 0, 1, 0 \rangle| \, dA$$

$$= |\langle 2x, 0, 1 \rangle| \, dA = \sqrt{4x^2 + 1} \, dA$$

Now continue as in Soln 1.