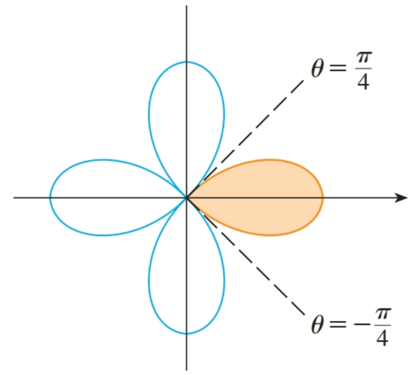
MA261-YOLCU PRACTICE PROBLEMS FOR TEST 2 SPRING 2013

(1) Consider $f(x, y) = x^4 + y^4 - 4xy + 1$. Show that f has a local minimum at $(1, 1)$ and $(-1, -1)$ and that $(0, 0)$ is a saddle point of f.

(2) Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$. Answer: $f(0, \pm 1) = 2$ is the maximum, $f(\pm 1, 0) = 1$ is the minimum value.

(3) Find the area of **one loop** of the rose $r = \cos(2\theta)$ sketched below. Answer: $\frac{\pi}{8}$



(4) Find the value of the integral $I = \int_0^{\sqrt{2}} \int_{y^2}^{2} y\, e^{x^2}\, dx\, dy$ by interchanging the order of integration.

Answer:

$$I = \int_0^2 \int_0^{\sqrt{x}} y\, e^{x^2}\, dy\, dx = \frac{1}{4}(e^4 - 1).$$

See also similar problems on page: 996 (15.3(#49 − 54)).

(5) Use the midpoint rule with $m = n = 2$ to approximate

$$\iint_R (x^2 - 1)y \, dA$$

where R is the region $\{(x, y) \ : \ 0 \leq x \leq 4, \ \ 2 \leq y \leq 4\}$.

Asnwer: 96

(6) Let R be the region in the first quadrant bounded by $x = 0$, $x - y = 0$, $x^2 + y^2 = 9$ and $x + y = 6$. Evaluate

$$\iint_R \frac{x + y}{x^2 + y^2} \, dA.$$

Answer: $\dfrac{3}{2}\pi - 3$

(7) Find the center of mass $(\bar{x}, \bar{y})$ of the semicircular lamina described by $\{(x, y) \ : \ x^2 + y^2 \leq a^2, \ y \geq 0\}$ if its density at the point (x, y) is $\rho(x, y) = \sqrt{x^2 + y^2}$.

Answer: $\bar{x} = 0, \ \ \bar{y} = \dfrac{3a}{2\pi}$

(8) Find the area of the region described by the intersection of two disks bounded by $x^2+y^2 = x$ and $x^2+y^2 = y$.

Answer: $\dfrac{\pi}{8} - \dfrac{1}{4}$

(9) Find a, b, c, d, e, f, g, h so that

$$\int_0^1 \int_0^y \int_{\sqrt{y}}^1 F(x,y,z)\,dx\,dz\,dy = \int_0^1 \int_a^b \int_c^d F(x,y,z)\,dy\,dx\,dz = \int_0^1 \int_e^f \int_g^h F(x,y,z)\,dz\,dy\,dx$$

Answer: $a = \sqrt{z},\ b = 1,\ c = z,\ d = x^2,\ e = 0, f = x^2,\ g = 0,\ h = y.$

(10) Find the volume the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ below the sphere $x^2 + y^2 + z^2 = 2z$.

Answer: π

(11) Let T be the solid region in the first octant that is bounded by the planes $y = 2$, $x = 0$, $y = 2x$, $z = 0$, and $z = 2y$. What is the value of the tripple integral $\iiint_T x\,dV$?

Answer: 1

(12) Let c be a constant such that $0 < c < \pi/2$ or $\pi/2 < c < \pi$. Show that the equation of the surface $\phi = c$ converted to rectangular coordinates becomes $z = \cot(c)\sqrt{x^2 + y^2}$.

(13) A lamina L occupies the triangular region in the $xy-$plane with vertices $(0,0)$, $(0,3)$ and $(3,3)$. If the mass density at (x,y) is $\rho(x,y) = x + 2y$, then show that the $y-$coordinate of the center of mass of L is equal to $\frac{9}{4}$.

(14) Let E be the solid region enclosed by the paraboloid $z = x^2 + y^2$ and the plane $2x + z = 4$. Find the triple integral in cylindrical coordinates that gives the volume $V(E)$ of solid E.

Answer:

$$V(E) = \int_0^{2\pi} \int_0^{-\cos\theta + \sqrt{4+\cos^2\theta}} \int_{r^2}^{4-2r\cos\theta} r\, dz\, dr\, d\theta$$

(15) Convert

$$\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{\sqrt{x^2+y^2}} x\,y\,z\,dz\,dy\,dx$$

to spherical coordinates.

Answer:

$$\int_{-\pi/2}^{\pi/2} \int_{\pi/4}^{\pi/2} \int_0^{3/\sin\phi} \rho^5 \cos\phi\,\sin^3\phi\,\cos\theta\,\sin\theta\,d\rho\,d\phi\,d\theta$$

(16) What is the value of $\displaystyle\int_{-2}^{2}\int_0^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,dz\,dx\,dy$?

Answer: $\dfrac{64\pi}{9}$.

(17) Evaluate $\iiint_E \sqrt{x^2+z^2}\,dV$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.

Answer: $\dfrac{128\pi}{15}$.

(18) Find the surface area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 9$.

Answer: $\dfrac{\pi}{6}(37\sqrt{37} - 1)$.

(19) Find K, L, M, and N so that

$$\int_0^2 \int_{1-(x/2)}^{1-(x^2/4)} f(x,y)\; dy\, dx = \int_K^L \int_M^N f(x,y)\; dx\, dy.$$

Answer: $K = 0,\;\; L = 1,\;\; M = 2 - 2y,\;\; N = 2\sqrt{1-y}.$

(20) Write the double integral in polar coordinates representing the area of the planar region bounded on the right by $x^2 + y^2 = 2$ and bounded on the left by $x = 1$.

Answer: $\displaystyle\int_{-\pi/4}^{\pi/4} \int_{\sec\theta}^{\sqrt{2}} r\; dr\, d\theta = \frac{\pi}{2} - 1.$