Spring 2012

MA 266

Review Topics - Exam # 2

(1) **First Order Differential Equations.** (Separable, 1^{st} Order Linear, Homogeneous, Exact)

(2) **Second Order Linear Homogeneous with Equations Constant Coefficients** .

The differential equation $ay'' + by' + cy = 0$ has *Characteristic Equation* $ar^2 + br + c = 0$. Call the roots r_1 and r_2. The general solution of $ay'' + by' + cy = 0$ is as follows:

(a) If r_1, r_2 are real and distinct $\Rightarrow$ $y = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

(b) If $r_1 = \lambda + i\mu$ (hence $r_2 = \lambda - i\mu$) $\Rightarrow$ $y = C_1 e^{\lambda t}\cos \mu t + C_2 e^{\lambda t}\sin \mu t$

(c) If $r_1 = r_2$ (repeated roots) $\Rightarrow$ $y = C_1 e^{r_1 t} + C_2 t e^{r_1 t}$

(3) **Theory of 2^{nd} Linear Order Equations**.

Wronskian of y_1, y_2 is $W(y_1, y_2)(t) = \begin{vmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{vmatrix}$.

(a) The functions $y_1(t)$ and $y_2(t)$ are linearly independent over $a < t < b$ if $W(y_1, y_2) \neq 0$ for at least one point in the interval.

(b) **THEOREM (Existence & Uniqueness)** If $p(t), q(t)$ and $g(t)$ are continuous in an open interval $\alpha < t < \beta$ containing t_0, then the IVP $\begin{cases} y'' + p(t)\,y' + q(t)\,y = g(t) \\ y(t_0) = y_0 \\ y'(t_0) = y_1 \end{cases}$ has a unique solution $y = \phi(t)$ defined in the open interval $\alpha < t < \beta$.

(c) **Superposition Principle** If $y_1(t)$ and $y_2(t)$ are solutions of the 2^{nd} order linear homogeneous equation $P(t)y'' + Q(t)y' + R(t)y = 0$ over the interval $a < t < b$, then $y = C_1\, y_1(t) + C_2\, y_2(t)$ is also a solution for any constants C_1 and C_2.

(d) **THEOREM (Homogeneous)** If $y_1(t)$ and $y_2(t)$ are solutions of the linear homogeneous equation $P(t)y'' + Q(t)y' + R(t)y = 0$ in some interval I and $W(y_1, y_2) \neq 0$ for some t_1 in I, then the general solution is $y_c(t) = C_1\, y_1(t) + C_2\, y_2(t)$. This is usually called the *complementary solution* and we say that $y_1(t), y_2(t)$ form a *Fundamental Set of Solutions* (FSS) to the differential equation.

(e) **THEOREM (Nonhomogeneous)** The general solution of the nonhomogeneous equation

$$P(t)y'' + Q(t)y' + R(t)y = G(t)$$

is $y(t) = y_c(t) + y_p(t)$, where $y_c(t) = C_1\, y_1(t) + C_2\, y_2(t)$ is the general solution of the corresponding homogeneous equation $P(t)y'' + Q(t)y' + R(t)y = 0$ and $y_p(t)$ is a particular solution of the nonhomogeneous equation $P(t)y'' + Q(t)y' + R(t)y = G(t)$.

(f) **Useful Remark** : If $y_{p_1}(t)$ is a particular solution of $P(t)y'' + Q(t)y' + R(t)y = G_1(t)$ and if $y_{p_2}(t)$ is a particular solution of $P(t)y'' + Q(t)y' + R(t)y = G_2(t)$, then

$$y_p(t) = y_{p_1}(t) + y_{p_2}(t)$$

is a particular solution of $P(t)y'' + Q(t)y' + R(t)y = [G_1(t) + G_2(t)]$.

(4) **Reduction of Order**. If $y_1(t)$ is one solution of $P(t)y'' + Q(t)y' + R(t)y = 0$, then a second solution may be obtained using the substitution $\boxed{\mathbf{y = v(t)\, y_1(t)}}$. This reduces the original 2^{nd} order equation to a 1^{st} equation using the substitution $\mathrm{w} = \dfrac{\mathrm{dv}}{\mathrm{dt}}$. Solve that first order equation for w, then since $\mathrm{w} = \dfrac{\mathrm{dv}}{\mathrm{dt}}$, solve this 1^{st} order equation to determine the function v.

(5) **Finding A Particular Solution $y_p(t)$ to Nonhomogeneous Equations**.

You can always use the method of Variation of Parameters to find a particular solution $y_p(t)$ of the linear nonhomogeneous equation $y'' + p(t)\,y' + q(t)\,y = g(t)$. Variation of Parameters may require integration techniques.

If the coefficients of the differential equation are constants rather than functions **and** if $g(t)$ has a very special form (see table below), it is usually easier to use Undetermined Coefficients :

(a) UNDETERMINED COEFFICIENTS - **IF** $ay'' + by' + cy = g(t)$ **AND** $g(t)$ is as below:

$\boldsymbol{g(t)}$	Form of $\boldsymbol{y_p(t)}$
$P_m(t) = a_m t^m + a_{m-1} t^{m-1} + \cdots + a_0$	$t^s\,\{A_m t^m + A_{m-1} t^{m-1} + \cdots + A_0\}$
$e^{\alpha t}\, P_m(t)$	$t^s\,\{e^{\alpha t}\,(A_m t^m + A_{m-1} t^{m-1} + \cdots + A_0)\}$
$e^{\alpha t}\, P_m(t)\, \cos\beta t$ or $e^{\alpha t}\, P_m(t)\, \sin\beta t$	$t^s\,\{e^{\alpha t}\,[F_m(t)\,\cos\beta t + G_m(t)\,\sin\beta t]\}$

where $s =$ the smallest nonnegative integer ($s = 0, 1$ or 2) such that no term of $y_p(t)$ is a solution of the corresponding homogeneous equation. In other words, no term of $y_p(t)$ is a term of $y_c(t)$. ($F_m(t)$, $G_m(t)$ are both polynomials of degree m.)

(b) VARIATION OF PARAMETERS - If $y_1(t)$ and $y_2(t)$ are two independent solutions of the homogeneous equation $y'' + p(t)\,y' + q(t)\,y = 0$, then a particular solution $y_p(t)$ of the nonhomogeneous equation

$$y'' + p(t)\,y' + q(t)\,y = g(t) \quad (*)$$

has the form

$$y_p(t) = u_1(t)\,y_1(t) + u_2(t)\,y_2(t)$$

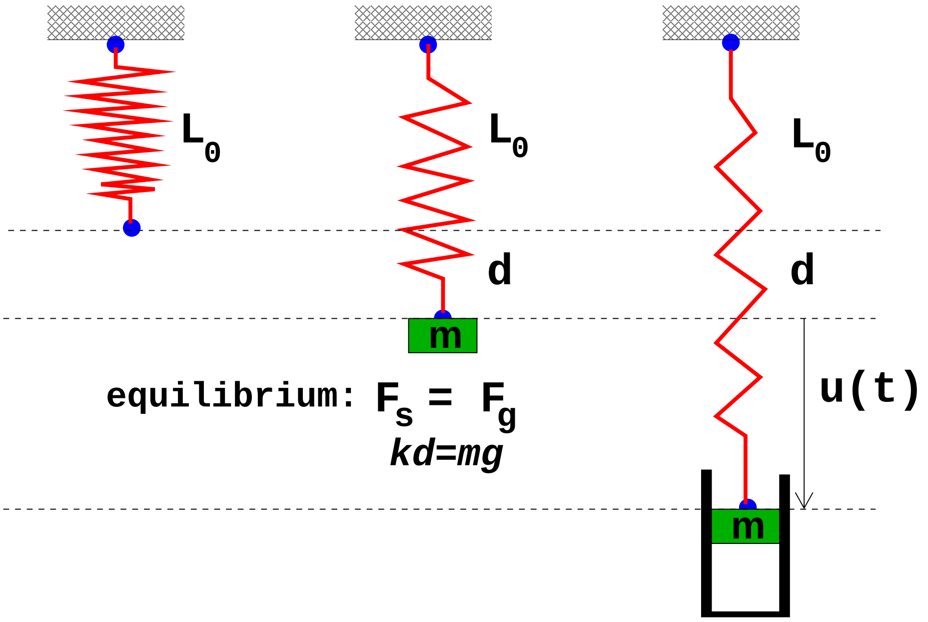
where

$$u_1' = \frac{\begin{vmatrix} 0 & y_2 \\ g(t) & y_2' \end{vmatrix}}{\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}}, \quad u_2' = \frac{\begin{vmatrix} y_1 & 0 \\ y_1' & g(t) \end{vmatrix}}{\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}}.$$

Remember: Coefficient of y'' in $(*)$ must be "1" in order to use the above formulas.

(6) Spring-Mass Systems $\begin{cases} m\,u'' + \gamma\,u' + k\,u = F(t) \\ u(0) = u_0\,,\ u'(0) = u_1 \end{cases}$

$m =$ mass of object, $\gamma =$ damping constant, $k =$ spring constant, $F(t) =$ external force
Weight $w = m\,g$, Hooke's Law : $F_s = k\,d$,



I Undamped Free Vibrations : $m\,u'' + k\,u = 0$ (Simple Harmonic Motion)
Note that $A\cos\omega_0 t + B\sin\omega_0 t = R\cos(\omega_0 t - \delta)$, where $R = \sqrt{A^2+B^2} =$ *amplitude*, $\omega_0 =$ *frequency*, $\dfrac{2\pi}{\omega_0} =$ *period* and $\delta =$ *phase shift* determined by $\tan\delta = \dfrac{B}{A}$.

II Damped Free Vibrations : $m\,u'' + \gamma\,u' + k\,u = 0$

(i) $\gamma^2 - 4km > 0$ (*overdamped*) $\Longleftrightarrow$ distinct real roots to CE

(ii) $\gamma^2 - 4km = 0$ (*critically damped*) $\Longleftrightarrow$ repeated roots to CE

(iii) $\gamma^2 - 4km < 0$ (*underdamped*) $\Longleftrightarrow$ complex roots to CE (motion is *oscillatory*)

III Forced Vibrations : ($F(t) = F_0\,\cos\omega t$ or $F(t) = F_0\,\sin\omega t$, for example)

(i) $m\,u'' + \gamma\,u' + k\,u = F(t)$ (Damped) In this case if you write the general solution as $u(t) = u_T(t) + u_\infty(t)$, then $u_T(t) =$ *Transient Solution* (i.e. the part of $u(t)$ such that $u_T(t) \longrightarrow 0$ as $t \longrightarrow \infty$) and $u_\infty(t) =$ *Steady-State Solution* (the solution behaves like this function in the long run).

(ii) $m\,u'' + k\,u = F_0\,\cos\omega t$ (Undamped) If $\omega = \omega_0 = \sqrt{\dfrac{k}{m}} \Rightarrow$ Resonance occurs and the solution is unbounded; while if $\omega \neq \omega_0$ then motion is a series of *beats* (solution is bounded)

(7) nth Order Linear Homogeneous Equations With Constant Coefficients

$$a_0 y^{(n)} + a_1 y^{(n-1)} + \cdots + a_{n-1}y' + a_n y = 0 \quad (*)$$

This differential equation has $\boldsymbol{n}$ independent solutions.

Characteristic Equation : $a_0 r^n + a_1 r^{n-1} + \cdots + a_{n-1} r + a_n = 0$ will have $\boldsymbol{n}$ characteristic roots that may be real and distinct, repeated, complex, or complex and repeated.

(a) For each real root r that is not repeated $\Rightarrow$ get a solution of $(*)$: e^{rt}

(b) For each real root r that is repeated $\boldsymbol{m}$ times $\Rightarrow$ get $\boldsymbol{m}$ independent solutions of $(*)$:

$$e^{rt},\ te^{rt},\ t^2e^{rt},\ \cdots,\ t^{m-1}e^{rt}$$

(c) For each complex root $r = \lambda + i\mu$ repeated $\boldsymbol{m}$ times $\Rightarrow$ get $\boldsymbol{2m}$ solutions of $(*)$:

$e^{\lambda t}\cos\mu t,\ te^{\lambda t}\cos\mu t,\ \cdots,\ t^{m-1}e^{\lambda t}\cos\mu t$ and $e^{\lambda t}\sin\mu t,\ te^{\lambda t}\sin\mu t,\ \cdots,\ t^{m-1}e^{\lambda t}\sin\mu t$

(don't need to consider its conjugate root $\lambda - i\mu$)

(8) Undetermined Coefficients for n^{th} Order Linear Equations

This can only be used to find $y_p(t)$ of $a_0y^{(n)} + a_1y^{(n-1)} + \cdots + a_{n-1}y' + a_ny = g(t)$ and $g(t)$ one of the 3 very SPECIAL FORMS in table in **(5)** above. The particular solution has the same form as before : $y_p(t) = t^s[\cdots]$, where $s =$ the smallest nonnegative integer such that no term of $y_p(t)$ is a term of $y_c(t)$, except this time $s = 0, 1, 2, \ldots, n$.

(9) Laplace Transforms

(a) Be able to compute Laplace transforms using definition :

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\,dt$$

and using a table of Laplace transforms (see table on page 317) and using linearity : $\mathcal{L}\{f(t) + g(t)\} = \mathcal{L}\{f(t)\} + \mathcal{L}\{g(t)\}$, $\mathcal{L}\{c\,f(t)\} = c\,\mathcal{L}\{f(t)\}$.

(b) Computing Inverse Laplace Transforms: Must be able to use a table of Laplace transforms usually together with *Partial Fractions* or *Completing the Square*, to find inverse Laplace transforms: $f(t) = \mathcal{L}^{-1}\{F(s)\}$.

(c) Solving Initial Value Problems: Recall that

$$\mathcal{L}\{y'\} = s\,\mathcal{L}\{y\} - y(0)$$

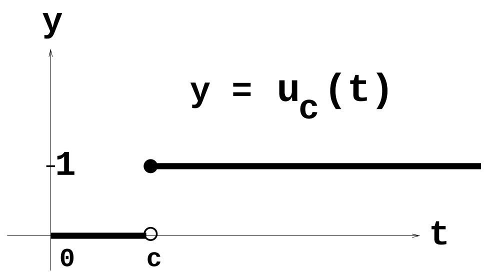
$$\mathcal{L}\{y''\} = s^2\,\mathcal{L}\{y\} - s\,y(0) - y'(0)$$

$$\mathcal{L}\{y'''\} = s^3\,\mathcal{L}\{y\} - s^2\,y(0) - s\,y'(0) - y''(0)$$

$$\vdots$$

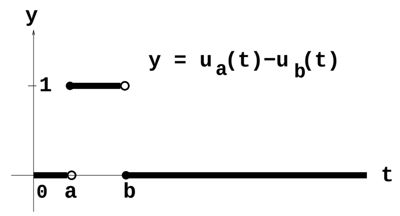
(d) Discontinuous Functions :

(i) **Unit Step Function** (Heaviside Function) : If $c \geq 0$, $u_c(t) = \begin{cases} 0, & t < c \\ 1, & t \geq c \end{cases}$

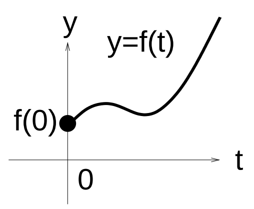


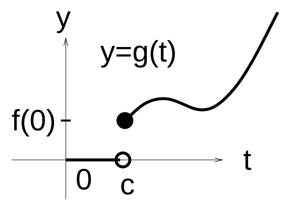
$$\mathcal{L}\{u_c(t)\} = \frac{e^{-cs}}{s}$$

(ii) **Unit "Pulse" Function** : $u_a(t) - u_b(t) = \begin{cases} 1, & a \le t < b \\ 0, & \text{otherwise} \end{cases}$



(iii) **Translated Functions**: $y = g(t) = \begin{cases} 0\,, & t < c \\ f(t-c)\,, & t \ge c \end{cases} = u_c(t)\,f(t-c)\,.$





$$\mathcal{L}\{u_c(t)\,f(t-c)\} = e^{-cs}\,F(s)\,, \text{where } F(s) = \mathcal{L}\{f(t)\}$$

Thus,
$$\mathcal{L}^{-1}\{e^{-cs}\,F(s)\} = u_c(t)\,f(t-c)\,, \quad \text{where } f(t) = \mathcal{L}^{-1}\{F(s)\}$$

A useful formula **NOT** in the book :
$$\boldsymbol{\mathcal{L}\{u_c(t)\,h(t)\} = e^{-cs}\,\mathcal{L}\{h(t+c)\}}$$

(iv) **Unit Impulse Functions**: If $y = \delta(t-c)$ $(\,c \ge 0)$, then
$$\mathcal{L}\,\{\delta(t-c)\} = e^{-cs}$$

(e) Convolutions:
$$\mathcal{L}\{(f*g)(t)\} = \mathcal{L}\left\{\int_0^t f(t-\tau)\,g(\tau)\,d\tau\right\} = \mathcal{L}\{f(t)\}\;\mathcal{L}\{g(t)\}$$

Practice Problems

[1] For what value of α will the solution to the IVP $\begin{cases} y'' - y' - 2y = 0 \\ y(0) = \alpha \\ y'(0) = 2 \end{cases}$ satisfy $y \to 0$ as $t \to \infty$?

[2] (a) Show that $y_1 = x$ and $y_2 = x^{-1}$ are solutions of the differential equation $x^2y'' + xy' - y = 0$.
(b) Evaluate the Wronskian $W(y_2, y_1)$ at $x = \frac{1}{2}$.
(c) Find the solution of the initial value problem $x^2y'' + xy' - y = 0,\ y(1) = 2,\ y'(1) = 4$.

[3] Find the largest open interval for which the initial value problem

$$3x^2y'' + y' + \frac{1}{x-2}y = \frac{1}{x-3},\ y(1) = 3,\ y'(1) = 2, \text{ has a solution.}$$

In Problems 4, 5, and 6 find the general solution of the homogeneous differential equations in (a) and use the method of **Undetermined Coefficients** to find a particular solution y_p in (b) and find the FORM of a particular solution (c).

[4] (a) $y'' - 5y' + 6y = 0$ (b) $y'' - 5y' + 6y = t^2$ (c) $y'' - 5y' + 6y = e^{2t} + \cos(3t)$

[5] (a) $y'' - 6y' + 9y = 0$ (b) $y'' - 6y' + 9y = te^{3t}$ (c) $y'' - 6y' + 9y = e^t + \cos(3t)$

[6] (a) $y'' - 2y' + 10y = 0$ (b) $y'' - 2y' + 10y = e^x + \cos(3x)$ (c) $y'' - 2y' + 10y = e^x\cos(3x)$

[7] Find the general solution to (a) $y'' + y' - 6y = 7e^{4t}$ (b) $y'' + y' - 6y = 7e^{4t} - 100\sin t$

[8] Solve this IVP: $y'' - y' = 4t,\ y(0) = 0,\ y'(0) = 0$.

[9] Find the general solution to $y'' + y = \tan t\,,\ 0 < x < \frac{\pi}{2}$.

[10] The differential equation $x^2y'' - 2xy' + 2y = 0$ has solutions $y_1(x) = x$ and $y_2 = x^2$. Use the method of **Variation of Parameters** to find a solution of $x^2y'' - 2xy' + 2y = 2x^2$.

[11] The differential equation $x^2y'' + xy' - y = 0$ has one solution $y_1(x) = x$. Use the method of **Reduction of Order** to find a second (linearly independent) solution of $x^2y'' + xy' - y = 0$.

[12] For what nonnegative values of γ will the the solution of the initial value problem $u'' + \gamma u' + 4u = 0,\ u(0) = 4,\ u'(0) = 0$ *oscillate* ?

[13] (a) For what positive values of k does the solution of the initial value problem $2u'' + ku = 3\cos(2t),\ u(0) = 0,\ u'(0) = 0,$ become *unbounded* (Resonance) ?

(b) For what positive values of k does the solution of the initial value problem $2u'' + u' + ku = 3\cos(2t),\ u(0) = 0,\ u'(0) = 0,$ become *unbounded* (Resonance) ?

[14] Find the steady–state solution of the IVP $y'' + 4y' + 4y = \sin t,\ y(0) = 0,\ y'(0) = 0$.

[15] A 4-kg mass stretches a spring 0.392 m. If the mass is released from 1 m below the equilibrium position with a downward velocity of 10 m/sec, what is the maximum displacement ?

In Problems 16 and 17 find the general solution of the homogeneous differential equations in (a) and use the method of **Undetermined Coefficients** to find the FORM of a particular solution of the nonhomogeneous equation in (b).

[16] (a) $y''' - y' = 0$ (b) $y''' - y' = t + e^t$

[17] (a) $y''' - y'' - y' + y = 0$ (b) $y''' - y'' - y' + y = e^t + \cos t$

[18] Find the solution of the initial value problem $y''' - 2y'' + y' = 0,\ y(0) = 2,\ y'(0) = 0,\ y''(0) = 1$.

[19] Find the general solution of the differential equation $y''' + y' = t^2$.

[20] Find the general solution of $y'' + 4y' = -10\cos 2t$.

[21] Find a fundamental set of solutions of $y^{(5)} - 4y''' = 0$.

[22] Find the Laplace transform of these functions:
(a) $f(t) = 3 - e^{2t}$ (b) $g(t) = 100\,t^5$ (c) $h(t) = \cosh \pi t$ (d) $k(t) = -10t^3e^{5t}$

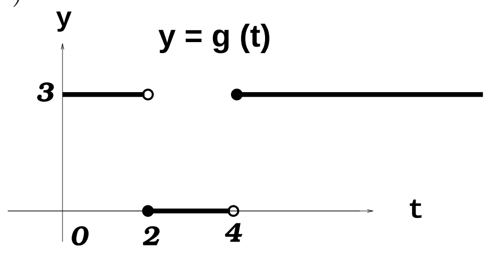
[23] Find the inverse Laplace transform of
(a) $F(s) = \dfrac{9}{s^2 - s - 2}$ (b) $F(s) = \dfrac{s}{(s-1)^2}$ (c) $F(s) = \dfrac{8}{(s+1)^4}$ (d) $F(s) = \dfrac{3s+2}{s^2 + 2s + 5}$

[24] Solve these initial value problems: (a) $\begin{cases} y'' - y' - 6y = 0 \\ y(0) = 1 \\ y'(0) = -1 \end{cases}$ (b) $\begin{cases} y'' - 2y' + 2y = \cos t \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$

(c) $y'' - y = \begin{cases} 1, & t < 5 \\ 2, & 5 \le t < \infty \end{cases}$; $y(0) = y'(0) = 0.$

(d) $y'' + 4y = \begin{cases} t\,, & t < 1 \\ 0\,, & 1 < t < \infty \end{cases}$; $y(0) = y'(0) = 0$.

(e) $y' + y = g(t)$, $y(0) = 0$ and where $g(t)$:



(f) $y'' + 4y = \delta(t-3)\,, \quad y(0) = y'(0) = 0$

[25] $\mathcal{L}\left\{\int_0^t 100\, e^{-2\tau} \cos \pi(t-\tau)\, d\tau\right\} = \,?$

[26] If $g(t) = \mathcal{L}^{-1}\{G(s)\}$, then $\mathcal{L}^{-1}\left\{\dfrac{G(s)}{(s-3)^2}\right\} = \,?$

Answers

[1] $\alpha = -2$ **[2]** (b) $W(x^{-1}, x)(\frac{1}{2}) = 4$; (c) $y = 3x - x^{-1}$ **[3]** $0 < x < 2$
[4] (a) $y = C_1 e^{2t} + C_2 e^{3t}$ (b) $y = At^2 + Bt + C$ (c) $y = Ate^{2t} + B\cos(3t) + C\sin(3t)$
[5] (a) $y = C_1 e^{3t} + C_2 te^{3t}$ (b) $y = t^2(At + B)e^{3t}$ (c) $y = Ae^t + B\cos(3t) + C\sin(3t)$
[6] (a) $y = C_1 e^x \cos(3x) + C_2 e^x \sin(3x)$ (b) $y = Ae^x + B\cos(3x) + C\sin(3x)$
(c) $y = x(A\cos(3x) + B\sin(3x))e^x$
[7](a) $y = C_1 e^{-3t} + C_2 e^{2t} + \frac{1}{2}e^{4t}$ (b) $y = C_1 e^{-3t} + C_2 e^{2t} + \frac{1}{2}e^{4t} + 2\cos t + 14\sin t$
[8] $y = -4 + 4e^t - 2t^2 - 4t$
[9] $y = C_1 \cos t + C_2 \sin t - (\cos t)\ln(\sec t + \tan t)$
[10] $y = 2x^2 \ln x$ or $y = 2x^2 \ln x + (C_1 x + C_2 x^2)$
[11] $y = x^{-1}$ or $y = Ax^{-1} + Bx,\ A \neq 0$
[12] $0 \le \gamma < 4$
[13] (a) $k = 8$ (resonance) (b) NO value of k, all solutions are bounded.
[14] $y = \frac{1}{25}(3\sin t - 4\cos t)$
[15] $u(t) = \cos 5t + 2\sin 5t = \sqrt{5}\cos(5t - \delta),\ \delta = \tan^{-1} 2 \approx 1.1$ Thus amplitude $= \sqrt{5}$.
[16] (a) $y = C_1 + C_2 e^{-t} + C_3 e^t$ (b) $y = t(At + B) + Cte^t$
[17] (a) $y = C_1 e^t + C_2 te^t + C_3 e^{-t}$ (b) $y = At^2 e^t + B\cos t + C\sin t$
[18] $y = 3 - e^t + te^t$
[19] $y = C_1 + C_2 \cos t + C_3 \sin t + \dfrac{1}{3}t^3 - 2t$
[20] $y = C_1 + C_2 e^{-4t} + \left(\frac{1}{2}\cos 2t - \sin 2t\right)$
[21] $\{1, t, t^2, e^{2t}, e^{-2t}\}$
[22] (a) $\dfrac{2s-6}{s^2-2s}$ (b) $\dfrac{12000}{s^6}$ (c) $\dfrac{s}{s^2-\pi^2}$ (d) $-\dfrac{60}{(s-5)^4}$
[23] (a) $3(e^{2t} - e^{-t})$ (b) $e^t + te^t$ (c) $\frac{4}{3}t^3 e^{-t}$ (d) $3e^{-t}\cos 2t - \frac{1}{2}\,e^{-t}\sin 2t$
[24] (a) $y = \frac{1}{5}(e^{3t} + 4e^{-2t})$ (b) $y = \frac{1}{5}(\cos t - 2\sin t + 4e^t\cos t - 2e^t \sin t)$
(c) $y = -1 + \frac{1}{2}(e^t + e^{-t}) + u_5(t)(-1 + \frac{1}{2}(e^{(t-5)} + e^{-(t-5)}))$,
or $y = -1 + \cosh t + u_5(t)(-1 + \cosh(t-5))$
(d) $y = (-\frac{1}{8}\sin 2t + \frac{t}{4}) - u_1(t)(-\frac{1}{8}\sin 2(t-1) + \frac{t-1}{4}) - u_1(t)(\frac{1}{4} - \frac{1}{4}\cos 2(t-1))$
(e) $y = 3\,(1 - e^{-t}) - 3\,u_2(t)(1 - e^{-(t-2)}) + 3\,u_4(t)(1 - e^{-(t-4)})$ (f) $y = \frac{1}{2}u_3(t)(t)\sin 2(t-3)$

[25] $\dfrac{100\,s}{(s+2)(s^2+\pi^2)}$ **[26]** $\displaystyle\int_0^t (t-\tau)\,e^{3(t-\tau)} g(\tau)\,d\tau$ or $\displaystyle\int_0^t \tau\, e^{3\tau} g(t-\tau)\,d\tau$
