## MA 271 PRACTICE PROBLEMS

1. If the line $\ell$ is parallel to the vector $\vec{v} = 2\vec{i} - 3\vec{j} + 7\vec{k}$, and contains the point $(2, 1, -3)$, then its vector equation is

   A. $\vec{r} = (1+2t)\vec{i} - 3t\vec{j} + (-2+7t)\vec{k}$ B. $\vec{r} = (2+t)\vec{i} - 3\vec{j} + (7-2t)\vec{k}$
   C. $\vec{r} = (2+2t)\vec{i} + (1-3t)\vec{j} + (-3+7t)\vec{k}$ D. $\vec{r} = (2+2t)\vec{i} + (-3+t)\vec{j} + (7-3t)\vec{k}$
   E. $\vec{r} = (2+t)\vec{i} + \vec{j} + (7-3t)\vec{k}$

2. Find parametric equations of the line containing the points $(1, -1, 0)$ and $(-2, 3, 5)$.

   A. $x = 1-3t, y = -1+4t, z = 5t$ B. $x = t, y = -t, z = 0$
   C. $x = 1-2t, y = -1+3t, z = 5t$ D. $x = -2t, y = 3t, z = 5t$
   E. $x = -1+t, y = 2-t, z = 5$

3. Find an equation of the plane that contains the point $(1, -1, -1)$ and has normal vector $\frac{1}{2}\vec{i} + 2\vec{j} + 3\vec{k}$.

   A. $x - y - z + \frac{9}{2} = 0$ B. $x + 4y + 6z + 9 = 0$ C. $\frac{x-1}{\frac{1}{2}} = \frac{y+1}{2} = \frac{z+1}{3}$
   D. $x - y - z = 0$ E. $\frac{1}{2}x + 2y + 3z = 1$

4. Find an equation of the plane that contains the points $(1, 0, -1)$, $(-5, 3, 2)$, and $(2, -1, 4)$.

   A. $6x - 11y + z = 5$ B. $6x + 11y + z = 5$ C. $11x - 6y + z = 0$
   D. $\vec{r} = 18\vec{i} - 33\vec{j} + 3\vec{k}$ E. $x - 6y - 11z = 12$

5. Find parametric equations of the line tangent to the curve $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$ at the point $(2, 4, 8)$

   A. $x = 2+t, y = 4+4t, z = 8+12t$ B. $x = 1+2t, y = 4+4t, z = 12+8t$
   C. $x = 2t, y = 4t, z = 8t$ D. $x = t, y = 4t, z = 12t$ E. $x = 2+t, y = 4+2t, z = 8+3t$

6. The position function of an object is
   $\vec{r}(t) = \cos t\vec{i} + 3\sin t\vec{j} - t^2\vec{k}$
   Find the velocity, acceleration, and speed of the object when $t = \pi$.

| | Velocity | Acceleration | Speed |
|---|---|---|---|
| A. | $-\vec{i} - \pi^2\vec{k}$ | $-3\vec{j} - 2\pi\vec{k}$ | $\sqrt{1+\pi^4}$ |
| B. | $\vec{i} - 3\vec{j} + 2\pi\vec{k}$ | $-\vec{i} - 2\vec{k}$ | $\sqrt{10+4\pi^2}$ |
| C. | $3\vec{j} - 2\pi\vec{k}$ | $-\vec{i} - 2\vec{k}$ | $\sqrt{9+4\pi^2}$ |
| D. | $-3\vec{j} - 2\pi\vec{k}$ | $\vec{i} - 2\vec{k}$ | $\sqrt{9+4\pi^2}$ |
| E. | $\vec{i} - 2\vec{k}$ | $-3\vec{j} - 2\pi\vec{k}$ | $\sqrt{5}$ |

7. A smooth parametrization of the semicircle which passes through the points $(1,0,5)$, $(0,1,5)$ and $(-1,0,5)$ is

A. $\vec{r}(t) = \sin t\vec{i} + \cos t\vec{j} + 5\vec{k}, 0 \le t \le \pi$  B. $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + 5\vec{k}, 0 \le t \le \pi$
C. $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + 5\vec{k}, \frac{\pi}{2} \le t \le \frac{3\pi}{2}$  D. $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + 5\vec{k}, 0 \le t \le \frac{\pi}{2}$
E. $\vec{r}(t) = \sin t + \cos t\vec{j} + 5\vec{k}, \frac{\pi}{2} \le t \le \frac{3\pi}{2}$

8. The length of the curve $\vec{r}(t) = \frac{2}{3}(1+t)^{\frac{3}{2}}\vec{i} + \frac{2}{3}(1-t)^{\frac{3}{2}}\vec{j} + t\vec{k}$, $-1 \le t \le 1$ is

A. $\sqrt{3}$  B. $\sqrt{2}$  C. $\frac{1}{2}\sqrt{3}$  D. $2\sqrt{3}$  E. $\sqrt{2}$

9. The level curves of the function $f(x,y) = \sqrt{1 - x^2 - 2y^2}$ are

A. circles  B. lines  C. parabolas  D. hyperbolas  E. ellipses

10. The level surface of the function $f(x,y,z) = z - x^2 - y^2$ that passes through the point $(1,2,-3)$ intersects the $(x,z)$-plane $(y = 0)$ along the curve

A. $z = x^2 + 8$  B. $z = x^2 - 8$  C. $z = x^2 + 5$  D. $z = -x^2 - 8$
E. does not intersect the $(x,z)$-plane

11. Match the graphs of the equations with their names:

| | |
|---|---|
| (1) $x^2 + y^2 + z^2 = 4$ | (a) paraboloid |
| (2) $x^2 + z^2 = 4$ | (b) sphere |
| (3) $x^2 + y^2 = z^2$ | (c) cylinder |
| (4) $x^2 + y^2 = z$ | (d) double cone |
| (5) $x^2 + 2y^2 + 3z^2 = 1$ | (e) ellipsoid |

A. 1b, 2c, 3d, 4a, 5e  B. 1b, 2c, 3a, 4d, 5e  C. 1e, 2c, 3d, 4a, 5b
D. 1b, 2d, 3a, 4c, 5e  E. 1d, 2a, 3b, 4e, 5c

12. Suppose that $w = u^2/v$ where $u = g_1(t)$ and $v = g_2(t)$ are differentiable functions of $t$. If $g_1(1) = 3$, $g_2(1) = 2$, $g_1'(1) = 5$ and $g_2'(1) = -4$, find $\frac{dw}{dt}$ when $t = 1$.

A. 6  B. 33/2  C. $-24$  D. 33  E. 24

13. If $w = e^{uv}$ and $u = r + s$, $v = rs$, find $\frac{\partial w}{\partial r}$.

A. $e^{(r+s)rs}(2rs + r^2)$  B. $e^{(r+s)rs}(2rs + s^2)$  C. $e^{(r+s)rs}(2rs + r^2)$
D. $e^{(r+s)rs}(1 + s)$  E. $e^{(r+s)rs}(r + s^2)$.

14. If $f(x,y)=\cos(xy)$, $\frac{\partial^2 f}{\partial x \partial y}=$

A. $-xy\cos(xy)$ B. $-xy\cos(xy)-\sin(xy)$ C. $-\sin(xy)$

D. $xy\cos(xy)+\sin(xy)$ E. $-\cos(xy)$

15. Assuming that the equation $xy^2+3z=\cos(z^2)$ defines $z$ implicitly as a function of $x$ and $y$, find $\frac{\partial z}{\partial x}$.

A. $\frac{y^2}{3-\sin(z^2)}$ B. $\frac{-y^2}{3+\sin(z^2)}$ C. $\frac{y^2}{3+2z\sin(z^2)}$ D. $\frac{-y^2}{3+2z\sin(z^2)}$ E. $\frac{-y^2}{3-2z\sin(z^2)}$

16. If $f(x,y)=xy^2$, then $\nabla f(2,3)=$

A. $12\vec{i}+9\vec{j}$ B. $18\vec{i}+18\vec{j}$ C. $9\vec{i}+12\vec{j}$ D. $21$ E. $\sqrt{2}$.

17. Find the directional derivative of $f(x,y)=5-4x^2-3y$ at $(x,y)$ towards the origin

A. $-8x-3$ B. $\frac{-8x^2-3y}{\sqrt{x^2+y^2}}$ C. $\frac{-8x-3}{\sqrt{64x^2+9}}$ D. $8x^2+3y$ E. $\frac{8x^2+3y}{\sqrt{x^2+y^2}}$.

18. For the function $f(x,y)=x^2y$, find a unit vector $\vec{u}$ for which the directional derivative $D_{\vec{u}}f(2,3)$ is zero.

A. $\vec{i}+3\vec{j}$ B. $\frac{i+3\vec{j}}{\sqrt{10}}$ C. $\vec{i}-3\vec{j}$ D. $\frac{i-3\vec{j}}{\sqrt{10}}$ E. $\frac{3\vec{i}-\vec{j}}{\sqrt{10}}$.

19. Find a vector pointing in the direction in which $f(x,y,z)=3xy-9xz^2+y$ increases most rapidly at the point $(1,1,0)$.

A. $3\vec{i}+4\vec{j}$ B. $\vec{i}+\vec{j}$ C. $4\vec{i}-3\vec{j}$ D. $2\vec{i}+\vec{k}$ E. $-\vec{i}+\vec{j}$.

20. Find a vector that is normal to the graph of the equation $2\cos(\pi xy)=1$ at the point $(\frac{1}{6},2)$.

A. $6\vec{i}+\vec{j}$ B. $-\sqrt{3}\,\vec{i}-\vec{j}$ C. $12\vec{i}+\vec{j}$ D. $\vec{j}$ E. $12\vec{i}-\vec{j}$.

21. Find an equation of the tangent plane to the surface $x^2+2y^2+3z^2=6$ at the point $(1,1,-1)$.

A. $-x+2y+3z=2$ B. $2x+4y-6z=6$ C. $x-2y+3z=-4$

D. $2x+4y-6z=0$ E. $x+2y-3z=6$.

22. Find an equation of the plane tangent to the graph of $f(x,y)=\pi+\sin(\pi x^2+2y)$ when $(x,y)=(2,\pi)$.

A. $4\pi x+2y-z=9\pi$ B. $4x+2\pi y-z=10\pi$ C. $4\pi x+2\pi y+z=10\pi$

D. $4x+2\pi y-z=9\pi$ E. $4\pi x+2y+z=9\pi$.

23. Let $f(x, y, z) = \dfrac{x^2y^4}{x^4 + 6y^8}$ when $x \neq 0$ and $f(0,0) = 0$. Which of the following are (is) true?
i) $\partial f/\partial x(0,0)$ and $\partial f/\partial y(0,0)$ exist at $(0,0)$.
ii) $f(x, y)$ is continuous at $(0,0)$.
iii) The graph of $z = f(x, y)$ has a tangent plane at $(0,0,0)$.

A. i) only B. i) and ii) only C. i) and iii) only D. i) ii), and iii) E. None

24. The function $f(x, y) = 2x^3 - 6xy - 3y^2$ has

A. a relative minimum and a saddle point
B. a relative maximum and a saddle point
C. a relative minimum and a relative maximum
D. two saddle points
E. two relative minima.

25. Consider the problem of finding the minimum value of the function $f(x, y) = 4x^2 + y^2$ on the curve $xy = 1$. In using the method of Lagrange multipliers, the value of $\lambda$ (even though it is not needed) will be

A. 2 B. $-2$ C. $\sqrt{2}$ D. $\frac{1}{\sqrt{2}}$ E. 4.

26. Evaluate the iterated integral $\int_1^3 \int_0^x \frac{1}{x}\, dydx$.

A. $-\frac{8}{9}$ B. 2 C. $\ln 3$ D. 0 E. $\ln 2$.

27. Consider the double integral, $\iint_R f(x, y)dA$, where $R$ is the portion of the disk $x^2 + y^2 \leq 1$, in the upper half-plane, $y \geq 0$. Express the integral as an iterated integral.

A. $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x, y)dydx$
B. $\int_{-1}^0 \int_0^{\sqrt{1-x^2}} f(x, y)dydx$
C. $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} f(x, y)dydx$
D. $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x, y)dydx$
E. $\int_0^1 \int_0^{\sqrt{1-x^2}} f(x, y)dydx$.

28. Find $a$ and $b$ for the correct interchange of order of integration:
$\int_0^2 \int_{x^2}^{2x} f(x, y)dydx = \int_0^4 \int_a^b f(x, y)dxdy$.

A. $a = y^2, b = 2y$
B. $a = \frac{y}{2}, b = \sqrt{y}$
C. $a = \frac{y}{2}, b = y$
D. $a = \sqrt{y}, b = \frac{y}{2}$
E. cannot be done without explicit knowledge of $f(x, y)$.

29. Evaluate the double integral $\iint_R y dA$, where $R$ is the region of the $(x, y)$-plane inside the triangle with vertices $(0,0)$, $(2,0)$ and $(2,1)$.

A. 2 B. $\frac{8}{3}$ C. $\frac{2}{3}$ D. 1 E. $\frac{1}{3}$.

30. The volume of the solid region in the first octant bounded above by the parabolic sheet $z = 1 - x^2$, below by the $xy$ plane, and on the sides by the planes $y = 0$ and $y = x$ is given by the double integral

A. $\int_0^1 \int_0^x (1 - x^2)dydx$
B. $\int_0^1 \int_0^{1-x^2} x\, dydx$
C. $\int_{-1}^1 \int_{-x}^x (1 - x^2)dydx$
D. $\int_0^1 \int_x^0 (1 - x^2)dydx$
E. $\int_0^1 \int_x^{1-x^2} dydx$.

31. The area of one leaf of the three-leaved rose bounded by the graph of $r = 5\sin 3\theta$ is

A. $\frac{5\pi}{6}$ B. $\frac{25\pi}{12}$ C. $\frac{25\pi}{6}$ D. $\frac{5\pi}{3}$ E. $\frac{25\pi}{3}$.

32. Find the area of the portion of the plane $x + 3y + 2z = 6$ that lies in the first octant.

A. $3\sqrt{11}$ B. $6\sqrt{7}$ C. $6\sqrt{14}$ D. $3\sqrt{14}$ E. $6\sqrt{11}$.

33. A solid region in the first octant is bounded by the surfaces $z = y^2$, $y = x$, $y = 0$, $z = 0$ and $x = 4$. The volume of the region is

A. 64 B. $\frac{64}{3}$ C. $\frac{32}{3}$ D. 32 E. $\frac{16}{3}$.

34. An object occupies the region bounded above by the sphere $x^2 + y^2 + z^2 = 32$ and below by the cone $z = \sqrt{x^2 + y^2}$. The mass density at any point of the object is equal to its distance from the $xy$ plane. Set up a triple integral in rectangular coordinates for the total mass $m$ of the object.

A. $\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$ B. $\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

C. $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$ D. $\int_{0}^{4} \int_{0}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

E. $\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} xy\,dz\,dy\,dx$.

35. Do problem 34 in spherical coordinates.

A. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho^3 \cos\varphi \sin\varphi\,d\rho\,d\varphi\,d\theta$ B. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho \cos\varphi \sin\varphi\,d\rho\,d\varphi\,d\theta$

C. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho^3 \sin^2\varphi\,d\rho\,d\varphi\,d\theta$ D. $\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^{\sqrt{32}} \rho^3 \cos\varphi \sin\varphi\,d\rho\,d\varphi\,d\theta$

E. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho \cos\varphi\,d\rho\,d\varphi\,d\theta$.

36. The double integral $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2(x^2+y^2)^3 dydx$ when converted to polar coordinates becomes

A. $\int_0^{\pi} \int_0^1 r^9 \sin^2\theta\,dr\,d\theta$ B. $\int_0^{\frac{\pi}{2}} \int_0^1 r^8 \sin^2\theta\,dr\,d\theta$ C. $\int_0^{\pi} \int_0^1 r^8 \sin\theta\,dr\,d\theta$

D. $\int_0^{\frac{\pi}{2}} \int_0^1 r^8 \sin\theta\,dr\,d\theta$ E. $\int_0^{\frac{\pi}{2}} \int_0^1 r^9 \sin^2\theta\,dr\,d\theta$.

37. Which of the triple integrals converts

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} dz\,dy\,dx$$

from rectangular to cylindrical coordinates?

A. $\int_0^{\pi} \int_0^2 \int_r^2 r\,dz\,dr\,d\theta$ B. $\int_0^{2\pi} \int_0^2 \int_r^2 r\,dz\,dr\,d\theta$ C. $\int_0^{2\pi} \int_{-2}^2 \int_r^2 r\,dz\,dr\,d\theta$

D. $\int_0^{\pi} \int_0^2 \int_r^2 r\,dz\,dr\,d\theta$ E. $\int_0^{\frac{2\pi}{2}} \int_{-2}^2 \int_r^2 r\,dz\,dr\,d\theta$.

38. If $D$ is the solid region above the $xy$-plane that is between $z = \sqrt{4 - x^2 - y^2}$ and $z = \sqrt{1 - x^2 - y^2}$, then $\iiint_D \sqrt{x^2 + y^2 + z^2}\ dV =$

A. $\frac{14\pi}{3}$ B. $\frac{16\pi}{3}$ C. $\frac{15\pi}{2}$ D. $8\pi$ E. $15\pi$.

39. Determine which of the vector fields below are conservative, i. e. $\vec{F} = \text{grad } f$ for some function $f$.

   1. $\vec{F}(x, y) = (xy^2 + x)\vec{i} + (x^2y - y^2)\vec{j}$.
   2. $\vec{F}(x, y) = \frac{x}{y}\,\vec{i} + \frac{y}{x}\,\vec{j}$.
   3. $\vec{F}(x, y, z) = ye^z\,\vec{i} + (xe^z + e^y)\vec{j} + (xy + 1)e^z\,\vec{k}$.

A. 1 and 2　　B. 1 and 3　　C. 2 and 3　　D. 1 only　　E. all three

40. Let $\vec{F}$ be any vector field whose components have continuous partial derivatives up to second order, let $f$ be any real valued function with continuous partial derivatives up to second order, and let $\nabla = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z}$. Find the incorrect statement.

A. $\text{curl}(\text{grad } f) = \vec{0}$　　B. $\text{div}(\text{curl } \vec{F}) = 0$　　C. $\text{grad}(\text{div } \vec{F}) = 0$

D. $\text{curl } \vec{F} = \nabla \times \vec{F}$　　E. $\text{div } \vec{F} = \nabla \cdot \vec{F}$

41. A wire lies on the $xy$-plane along the curve $y = x^2$, $0 \le x \le 2$. The mass density (per unit length) at any point $(x, y)$ of the wire is equal to $x$. The mass of the wire is

A. $(17\sqrt{17} - 1)/12$　　B. $(17\sqrt{17} - 1)/8$　　C. $17\sqrt{17} - 1$

D. $(\sqrt{17} - 1)/3$　　E. $(\sqrt{17} - 1)/12$

42. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = y\vec{i} + x^2\vec{j}$ and $C$ is composed of the line segments from $(0, 0)$ to $(1, 0)$ and from $(1, 0)$ to $(1, 2)$.

A. 0　　B. $\frac{2}{3}$　　C. $\frac{5}{6}$　　D. 2　　E. 3

43. Evaluate the line integral

$$\int_C x\,dx + y\,dy + xy\,dz$$

where $C$ is parametrized by $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + \cos t\vec{k}$ for $-\frac{\pi}{2} \le t \le 0$.

A. 1　　B. $-1$　　C. $\frac{1}{3}$　　D. $-\frac{1}{3}$　　E. 0

44. Are the following statements true or false?
   1. The line integral $\int_C (x^3 + 2xy)dx + (x^2 - y^2)dy$ is independent of path in the $xy$-plane.
   2. $\int_C (x^3 + 2xy)dx + (x^2 - y^2)dy = 0$ for every closed oriented curve $C$ in the $xy$-plane.
   3. There is a function $f(x, y)$ defined in the $xy$-plane, such that
      $\text{grad } f(x, y) = (x^3 + 2xy)\vec{i} + (x^2 - y^2)\vec{j}$.

A. all three are false　　B. 1 and 2 are false, 3 is true　　C. 1 and 2 are true, 3 is false

D. 1 is true, 2 and 3 are false　　E. all three are true

45. Evaluate $\int_C y^2 dx + 6xy\,dy$ where $C$ is the boundary curve of the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$, in the counterclockwise direction.

A. 0　　B. 4　　C. 8　　D. 16　　E. 32

46. If $C$ goes along the $x$-axis from $(0,0)$ to $(1,0)$, then along $y=\sqrt{1-x^2}$ to $(0,1)$, and then back to $(0,0)$ along the $y$-axis, then $\int_C\ xy\,dy =$

A. $-\int_0^1 \int_0^{\sqrt{1-x^2}} y\,dy\,dx$  B. $\int_0^1 \int_0^{\sqrt{1-x^2}} y\,dy\,dx$  C. $-\int_0^1 \int_0^{\sqrt{1-x^2}} x\,dy\,dx$
D. $\int_0^1 \int_0^{\sqrt{1-x^2}} x\,dy\,dx$  E. $0$

47. Evaluate $\int_C\ \vec{F}\cdot d\vec{r}$, if $\vec{F}(x,y) = (xy^2-1)\vec{i} + (x^2y-x)\vec{j}$ and $C$ is the circle of radius 1 centered at $(1,2)$ and oriented counterclockwise.

A. $2$  B. $\pi$  C. $0$  D. $-\pi$  E. $-2$

48. Green's theorem yields the following formula for the area of a simple region $R$ in terms of a line integral over the boundary $C$ of $R$, oriented counterclockwise. Area of $R = \iint_R dA =$

A. $-\int_C y\,dx$  B. $\int_C y\,dx$  C. $\int_C x\,dx$  D. $\frac{1}{2}\int_C y\,dx - x\,dy$  E. $-\int\ x\,dy$

49. Evaluate the surface integral $\iint_S x\,d\sigma$ where $S$ is the part of the plane $2x+y+z=4$ in the first octant.

A. $8\sqrt{6}$  B. $\frac{8}{3}\sqrt{6}$  C. $\frac{8}{3}\sqrt{14}$  D. $\frac{\sqrt{14}}{3}$  E. $\frac{\sqrt{10}}{3}$

50. If $S$ is the part of the paraboloid $z=x^2+y^2$ with $z\le 4$, $\vec{n}$ is the unit normal vector on $S$ directed upward, and $\vec{F}(x,y,z) = x\vec{i}+y\vec{j}+z\vec{k}$, then $\iint_S \vec{F}\cdot\vec{n}\,d\sigma =$

A. $0$  B. $8\pi$  C. $4\pi$  D. $-4\pi$  E. $-8\pi$

51. If $\vec{F}(x,y,z) = \cos z\vec{i} + \sin z\vec{j} + xy\vec{k}$, $S$ is the complete boundary of the rectangular solid region bounded by the planes $x=0$, $x=1$, $y=0$, $y=1$, $z=0$ and $z=\frac{\pi}{2}$, and $\vec{n}$ is the outward unit normal on $S$, then $\iint_S \vec{F}\cdot\vec{n}\,d\sigma =$

A. $0$  B. $\frac{1}{2}$  C. $1$  D. $\frac{\pi}{2}$  E. $2$

52. If $\vec{F}(x,y,z) = x\vec{i}+y\vec{j}+z\vec{k}$, $S$ is the unit sphere $x^2+y^2+z^2=1$ and $\vec{n}$ is the outward unit normal on $S$, then $\iint_S \vec{F}\cdot\vec{n}\,d\sigma =$

A. $-4\pi$  B. $\frac{2\pi}{3}$  C. $0$  D. $\frac{4\pi}{3}$  E. $4\pi$

53. Evaluate the limit $\lim_{n\to\infty}\left[1+\frac{(-1)^n}{n}\right]$.

A. 0 B. 1 C. $-1$ D. 2 E. limit does not exist

54. Evaluate the limit $\lim_{n\to\infty}(\sqrt[n]{n}+\frac{1}{n!})$.

A. 0 B. 1 C. $e$ D. $1/e$ E. limit does not exist

55. If $s=\sum_{n=0}^{\infty}\frac{2^{n+1}}{3^n}$, then $s=$

A. 3 B. 6 C. 9 D. 2 E. $4/3$

56. If $L=\sum_{n=1}^{\infty}\frac{1}{2^n}+\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n}$, then $L=$

A. $1/3$ B. $2/3$ C. 1 D. $4/3$ E. $5/3$

57. $\sum_{n=1}^{\infty}\frac{1}{(n^2+1)^p}$ converges when

A. $p>1$ B. $p\le 1$ C. $p\ge 1$ D. $p>\frac{1}{2}$ E. $p\le\frac{1}{2}$

58. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^p$ converges for

A. $p\le 1$ B. $p>1$ C. $p<0$ D. $p>0$ E. no values of $p$

59. Which of the following series converge conditionally?

$$\text{(i)}\ \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\qquad\text{(ii)}\ \sum_{n=2}^{\infty}\frac{(-1)^n n}{\ln n}\qquad\text{(iii)}\ \sum_{n=1}^{\infty}\frac{(-1)^n n}{e^n}$$

A. only (ii) B. only (i) and (iii) C. only (i) and (ii) D. all three E. none of them

60. Which of the following series converge?

$$\text{(i)}\ \sum_{n=1}^{\infty}\frac{(-1)^n}{n^{\frac{1}{4}}}\qquad\text{(ii)}\ \sum_{n=1}^{\infty}\frac{n!}{1\cdot 3\cdot 5\cdots(2n-1)}\qquad\text{(iii)}\ \sum_{n=1}^{\infty}\frac{4}{3}\left(\frac{1}{2}\right)^n$$

A. only (ii) B. only (i) and (iii) C. only (i) and (ii) D. all three E. none of them

61. The interval of convergence for the power series $\sum_{n=2}^{\infty}\frac{3^n x^n}{n\ln n}$ is

A. $-\frac{1}{3}\le x<\frac{1}{3}$ B. $-\frac{1}{3}<x\le\frac{1}{3}$ C. $0\le x\le\frac{1}{3}$ D. $-1\le x\le 2$ E. $-1<x<1$

62. Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty}\frac{nx^n}{2^n}$$

A. $-\frac{1}{2}<x<\frac{1}{2}$ B. $-2<x<2$ C. $-2\le x\le 2$ D. $-2<x\le 2$ E. $-\infty<x<\infty$

63. The fourth term of the Maclaurin series for $\frac{x^2+3}{x-1}$ is

A. $-x^3$ B. $3x^3$ C. $-3x^3$ D. $-4x^3$ E. $4x^3$

64. The first three nonzero terms of the Maclaurin series for $f(x)=(1-x^2)\sin x$ are

A. $x-\frac{5}{6}x^3+\frac{31}{150}x^5$ B. $1-\frac{3}{2}x^2+\frac{13}{24}x^4$ C. $x-\frac{7}{6}x^3+\frac{31}{150}x^5$ D . $x^2-\frac{7}{6}x^3+\frac{1}{25}x^5$ E. $x-\frac{7}{6}x^3+\frac{21}{120}x^5$

65. The fourth term of the Taylor series for $f(x) = \ln x$ centered at $a = 2$ is
A. $\frac{1}{6}(x-2)^3$ B. $\frac{1}{12}(x-2)^3$ C. $\frac{1}{24}(x-2)^3$ D. $-\frac{1}{3}(x-2)^3$ E. $-(x-2)^3$

**ANSWERS**

1–C, 2–A, 3–B, 4–B, 5–A, 6–D, 7–B, 8–D, 9–E, 10–B, 11–A, 12–E, 13–B, 14–B,

15–D, 16–C, 17–E 18–D, 19–A, 20–C, 21–E, 22–A, 23–A, 24–B, 25–E, 26–B, 27–C,

28–B, 29–E, 30–A, 31–B, 32–D, 33–B, 34–B, 35–A, 36–E, 37–B, 38–C, 39–B, 40–C,

41–A, 42–D, 43–D, 44–E, 45–D, 46–B, 47–D, 48–A, 49–B, 50–E, 51–A, 52–E, 53–B

54– B, 55–B, 56–E, 57–D, 58–E, 59–E, 60–D, 61–A, 62–B, 63–D, 64–E, 65–C