MA 35100

HW # 3 - due Monday, September 16

1. Let $A = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 2 & 0 & -4 & 0 \\ -1 & 0 & 2 & 1 \end{bmatrix}$.

(a) Does the vector $\mathbf{b} = \begin{bmatrix} 1 \\ 8 \\ -7 \end{bmatrix}$ belong to the span of the columns of A?

If so, write out the vector $\mathbf{b}$ as an explicit linear combination of columns of A.

(b) Does the vector $\mathbf{b} = \begin{bmatrix} 1 \\ 8 \\ 7 \end{bmatrix}$ belong to the span of the columns of A?

If so, write out the vector $\mathbf{b}$ as an explicit linear combination of columns of A.

(c) What condition on the constants a, b, c guarantees that the vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ will belong to the span of the columns of A?

<u>**Remark**</u>: Test your answer here with what you obtained in parts (a) and (b) above. Your answer in part (c) tells you precisely which vectors $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ belong to the span of the columns of A and those that do not.

(d) Does the vector $\mathbf{b} = \begin{bmatrix} 1 & 0 & -2 & 5 \end{bmatrix}$ belong to the span of the *rows* of A?

2. Which sets are **linearly independent** and which are **linearly dependent** ?

(a) $S = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$

(b) $S = \left\{ \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \begin{bmatrix} 2 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \right\}$

(c) $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} \right\}$

(d) $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$

(e) $S = \left\{ (1+x)^2, \, (1+x^2), \, (1-2x+x^2) \right\}$

(f) $S = \left\{ \sin^2 x, \; 2, \; 3\cos^2 x \right\}$.

3. <u>Page 108</u>: **T/F Question**: **# 2.2(a)(b)(c)**.

4. If $\mathbf{u}, \mathbf{w} \in \mathsf{Span}\left\{ \mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k \right\}$, show that $\mathbf{u} + \mathbf{w} \in \mathsf{Span}\left\{ \mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k \right\}$.