MA 35100

HW # 7 - due Friday, October 25

1. Which subsets W of V are *subspaces* of V:

(a) $W = \{A \in M(2,3) : \ \mathbf{Rank}\,(A) = 1\,\}$;

(b) $W = \mathbf{Span}\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \right\}$; $V = \mathbb{R}^2$

(c) $W = \{\, f(x) \in \mathcal{C}^2(\mathbb{R}) : \ f''(x) - xf(x) = 0 \,\}$; $\quad V = \mathcal{C}^2(\mathbb{R})$

(d) $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : \quad x + y + z = 100 \right\}$; $V = \mathbb{R}^3$

(e) $W = \left\{ p(x) = a + bx^2 + cx^3 \in \mathcal{P}_3 \ : \ p(1) = p''(1) \right\}$; $\quad V = \mathcal{P}_3$

2. Find a basis for the subspace $W \subset \mathbb{R}_3$ given by

$$W = \mathbf{Span}\left\{ [1\ \ 1\ \ 1],\ [0\ \ 0\ \ 0],\ [1\ \ -1\ \ 3],\ [2\ \ -1\ \ 5] \right\}$$

3. **<u>TRUE/FALSE</u>** Questions:

(a) If A is a 4×7 matrix then $\mathbf{Nullity}\,(A) \leq 3$.

(b) A basis for the subspace $W = \mathbf{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$ is $\mathcal{B} = \mathbf{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

(c) There is a set of 5 matrices that will span $M(2,3)$.

(d) If A is a 3×5 matrix, then $\mathsf{Nullity}\,(A) = \mathsf{Nullity}\,(A^t)$.

4. Let $A = \begin{bmatrix} 0 & 1 & 2 & 1 & 2 \\ 0 & -1 & -2 & 1 & -1 \\ 0 & 3 & 6 & 1 & 5 \end{bmatrix}$. Find $\mathbf{Null}\,(A)$, $\mathbf{Null}\,(A^t)$ and then a basis for each.