MA 35100

HW # 9 - due Monday, November 11

1. TRUE/FALSE Questions

(a) If A is a 4×4 matrix and $\textbf{Col}\,(A) = \textbf{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$,

then $\textbf{Rank}\,(A) = 2$.

(b) $\mathcal{B} = \left\{ (x+1), (x^2+x), (x^2-1) \right\}$ is a basis for $\mathcal{P}_2$.

(c) If A is a 3×7 matrix, then $\textbf{Nullity}\,(A) \geq 4$.

(d) $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \;:\; y = 8x \right\}$ is NOT a subspace.

(e) If $L : \mathbb{R}^3 \to \mathbb{R}^2$ is a **LT** with $L \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ and $L \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$,

then $L \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

(f) If $\mathbf{v} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\} \implies [\,\mathbf{v}\,]_{\mathcal{B}} = \begin{bmatrix} -5 \\ 12 \end{bmatrix}$.

2. Find $\left(\frac{1}{12}\,A\right)^{-1}$ if $A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 0 \\ 2 & 3 & 0 \end{bmatrix}$.

3. Let $T : \mathcal{P}_2 \longrightarrow \mathbb{R}^2$ be defined by

$$T(p(x)) = \begin{bmatrix} p(1) \\ p'(1) \end{bmatrix}, \quad \text{i.e.} \quad T(a + bx + cx^2) = \begin{bmatrix} (a+b+c) \\ (b+2c) \end{bmatrix}.$$

(i) Show that T is a linear transformation (**LT**).

(ii) Find the matrix representation M for T (Standard basis for $\mathcal{P}_2$ and $\mathbb{R}^2$.)

4. $\begin{vmatrix} 10 & 20 & 50 \\ 20 & 20 & 20 \\ 4 & 0 & -4 \end{vmatrix} = \;?$