Fall 2019

MA 35100

HW # 10 - due Monday, November 25

1. **Page 249:** #4.1(h)(j).

2. **Page 259:** #4.15.

3. If $A = \begin{bmatrix} a & a & 1 \\ 1 & 2 & b \\ 2 & 2 & 0 \end{bmatrix}$, for what scalars a, b is A singular?

4. Let A be an $n \times n$ matrix.

(a) If $\det(A - \lambda I) = 0$, for a scalar λ and $B = QAQ^{-1}$, show that $\det(B - \lambda I) = 0$,

(b) If $\det(A - \lambda I) = 0$, for a scalar λ, show that $\det(A^2 - \lambda^2 I) = 0$,

5. If A is a 2×2 matrix and $\mathbf{v}_1, \mathbf{v}_2$ are two linearly independent non-zero vectors in $\mathbb{R}^2$ satisfying $A\mathbf{v}_1 = 3\mathbf{v}_1$ and $A\mathbf{v}_2 = -4\mathbf{v}_2$, compute $\det A$.

6. Let $B = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ and let $T : M(2,2) \longrightarrow M(2,2)$ be the transformation given by $T(A) = AB$.

(a) Show T is linear.

(b) Compute the determinant of the matrix M for T.

7. If A and B are 2×2 matrices with $|A| = 3$ and $\det(B^2) = 8$, compute

$$\det\left\{ -5A^2 \left(\frac{1}{2}AB^T\right)^{-1} \operatorname{adj}(A) \right\}.$$

8. **Page 268:** #4.36.

9. Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be the (nonlinear) transformation $T(\rho, \phi, \theta) = \Big(x(\rho,\phi,\theta),\ y(\rho,\phi,\theta),\ z(\rho,\phi,\theta) \Big)$, where

$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases}$$

Compute the Jacobian determinant $\left| \dfrac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)} \right|$ of the transformation T.