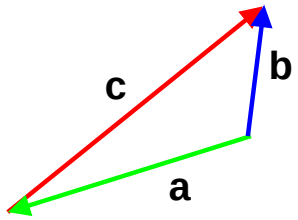
Homework Set # 1

1. **(§1.1) Page 18:** # 13, 16, 21, 37.

2. If $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^3$, prove the following:
   (a) *Parallelogram Law* : $||\mathbf{u}+\mathbf{v}||^2 + ||\mathbf{u}-\mathbf{v}||^2 = 2||\mathbf{u}||^2 + 2||\mathbf{v}||^2$
   (b) *Polarization Identity* : $||\mathbf{u}+\mathbf{v}||^2 - ||\mathbf{u}-\mathbf{v}||^2 = 4(\mathbf{u}\cdot\mathbf{v})$

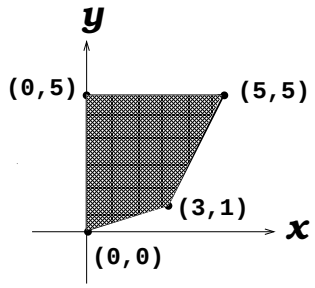
3. Prove the converse of the Pythagorean Theorem as follows:
   Suppose the sides of a triangle are the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ (as shown below). If $||\mathbf{a}||^2 + ||\mathbf{b}||^2 = ||\mathbf{c}||^2$, prove that $\mathbf{a}\cdot\mathbf{b} = 0$ (i.e., the triangle must be a right triangle).



4. **(§1.2) Page 29:** # 20, 26.

5. **(§1.3) Page 49:** # 4, 6, 11, 29, 33, 34.

6. Find the area of the polygon Ω shown here:



7. The line through the points $(3,2,1)$ and $(5,1,2)$ intersects the plane $x+y+z=14$ at what point?

8. The plane containing the points $P(1,1,1)$, $Q(2,0,-4)$, and $R(1,2,3)$ intersects the $x-$axis at what point?

9. Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$. Compute A^2 and A^3. Use mathematical induction to prove that $A^n = 3^{n-1}A$ for any positive integer n.