SOLUTIONS

MA 362 - Fall 2016

Homework Set # 2

1. **(§2.1) Page 85:** # 3(a), 4(a), 5(a), 6.

2. Sketch the graph of the function $z = f(x, y) = \dfrac{1}{\sqrt{x^2 + y^2}}$ using information from its level curves and various sections. This graph is sometimes called "Gabriel's Horn."

3. **(§2.1) Page 87:** # 29, 30.

Solutions

$\boxed{1}$ page 86 # 3(a): $f(x,y) = x^2 - y^2 = c \quad (c = 0, 1, -1)$

$f(x) = 0 \Leftrightarrow x^2 - y^2 = 0 \Leftrightarrow x^2 = y^2 \Leftrightarrow |x| = |y|$, so $\boxed{iii}$ ✓

$f(x) = 1 \Leftrightarrow x^2 - y^2 = 1$ is hyperbola opening along x-axis, so $\boxed{ii}$ ✓

$f(x) = -1 \Leftrightarrow x^2 - y^2 = -1$, $y^2 - x^2 = 1$ is hyperbola opening along y-axis

Thus $\boxed{i}$ ✓

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page 85 # 4(a): $f(x,y) = (x-y)^2 = c \quad (c = 0, 1, 4)$

$f(x) = 0 \Leftrightarrow (x-y)^2 = 0 \Leftrightarrow x = y \quad \therefore \boxed{iii}$ ✓

$f(x) = 1 \Leftrightarrow (x-y)^2 = 1 \Leftrightarrow |x-y| = 1$ so $(x-y) = 1$ or $(x-y) = -1$

$\therefore \; y = x - 1$ or $y = x + 1$

Hence $\boxed{ii}$

$f(x) = 4 \Leftrightarrow (x-y)^2 = 4 \Leftrightarrow |x-y| = 2$ and just as above $(x-y) = \pm 2$

so $y = x - 2$, or $y = x + 2$

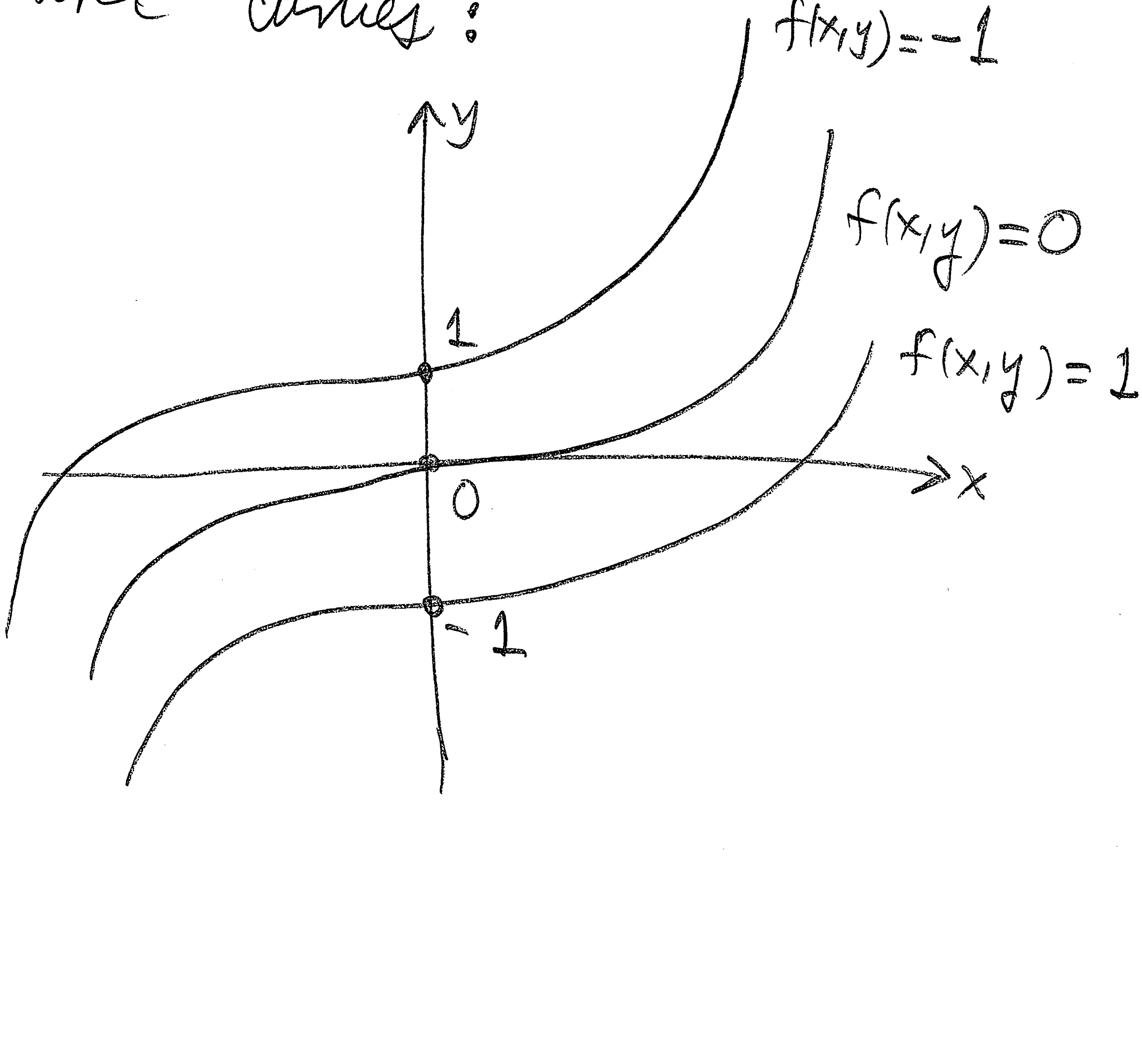
Hence $\boxed{i}$

page 85 #5(a): $f(x,y) = x^3 - y$

level curves are $f(x,y) = C$

so $x^3 - y = C \Rightarrow y = x^3 - C$

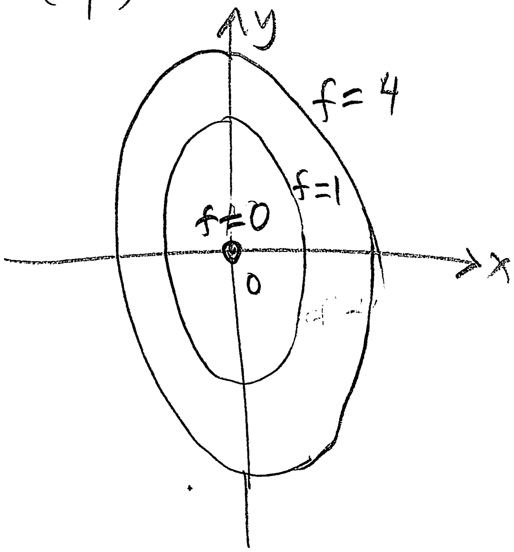
$\therefore$ When $C = -1, 0, 1$ we get these three curves:



page 85 #6: $z = f(x,y) = 9x^2 + y^2$

(a) level curves $f(x,y) = C \Rightarrow 9x^2 + y^2 = C$. Clearly $C \geq 0$ and level curve $f(x,y) = 0$ is the point $(0,0)$. Suppose $C > 0$, then

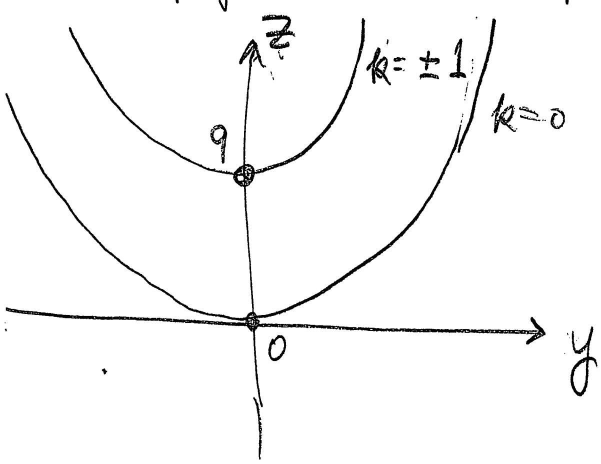
$\frac{x^2}{\left(\frac{C}{9}\right)} + \frac{y^2}{C} = 1$ are ellipses with major axis along y-axis.



(b) sections of graph of f in planes $x=-1$, $x=0$, $x=1$

$z = 9x^2 + y^2$

$\{x = k\} \cap \text{graph } f$ is $z = 9k^2 + y^2$

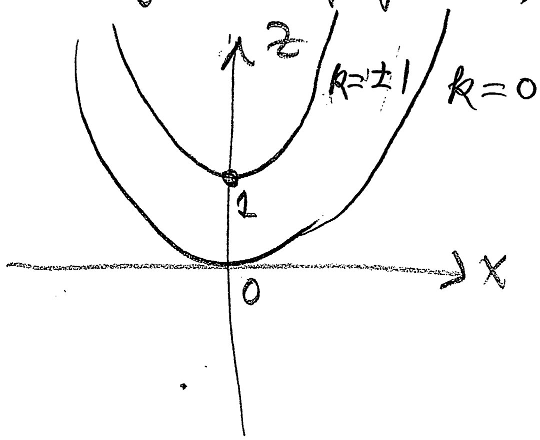


(cont'd)

Page 85 #6 (cont'd)

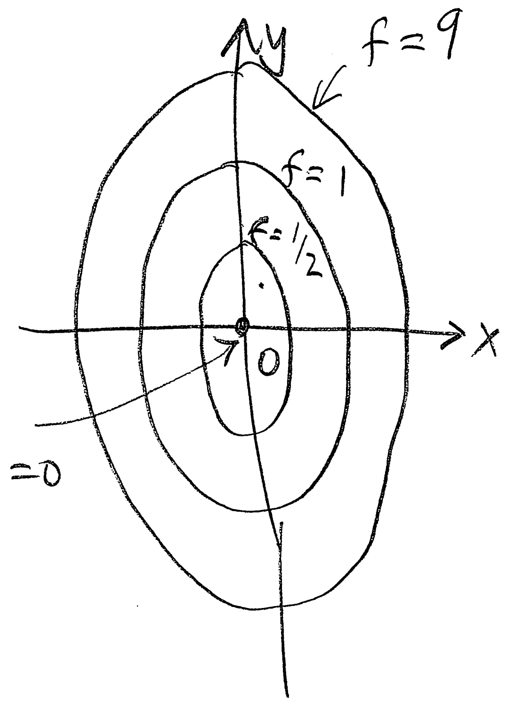
(c) Sections in planes $y=-1, y=0, y=1$: $z=9x^2+y^2$

so $\{y=k\} \cap \text{graph } f$ is $z=9x^2+k^2$



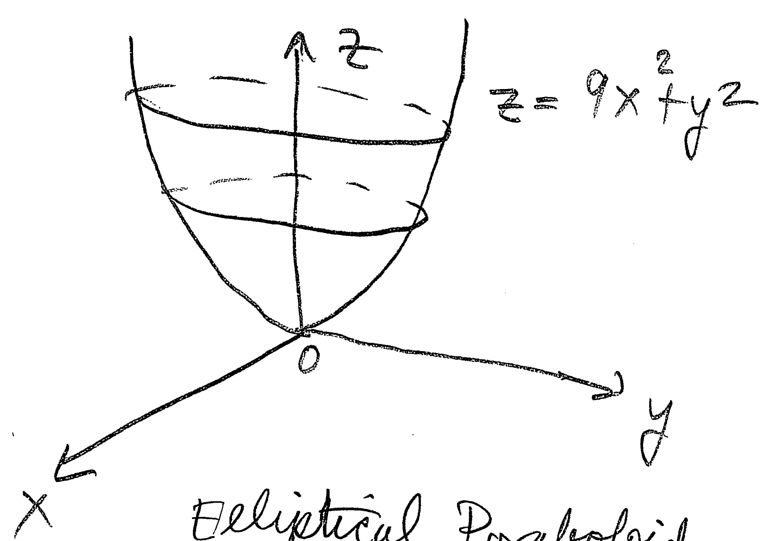
d) Graph of $z=f(x,y)=9x^2+y^2$

Level curves



$\Longrightarrow$

Graph of f



Elliptical Paraboloid

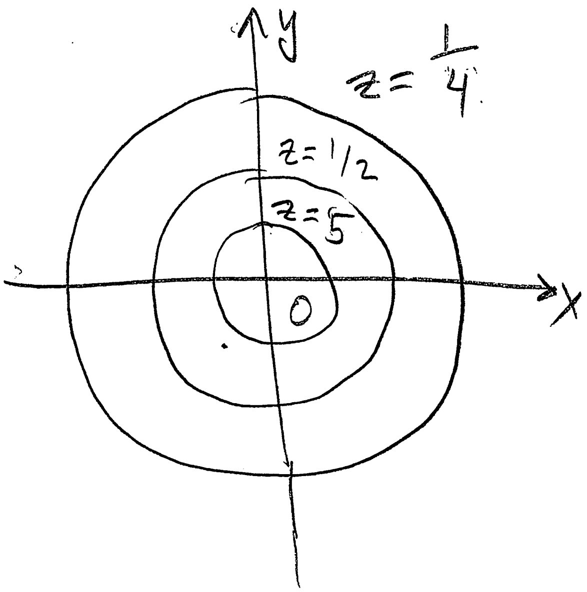
$\boxed{2}$ $z = f(x,y) = \dfrac{1}{\sqrt{x^2+y^2}}$: level curves $\dfrac{1}{\sqrt{x^2+y^2}} = C$

(note $0 < C < \infty$)

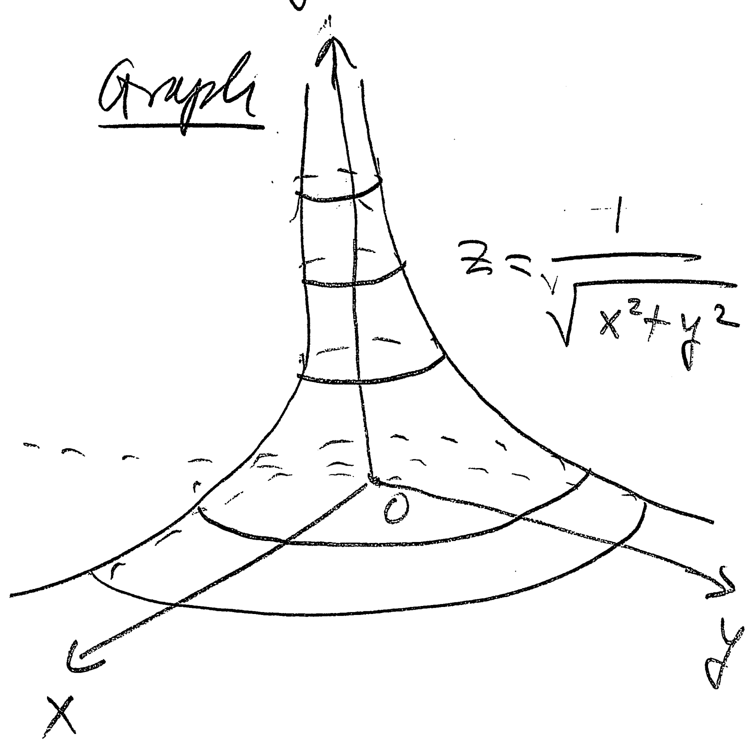
$x^2+y^2 = \dfrac{1}{C^2}$ circles with center $(0,0)$, radius $\dfrac{1}{C}$.

Section. $\{x=0\} \cap$ graph f is $z = \dfrac{1}{\sqrt{y^2}} = \dfrac{1}{|y|}$. Thus,

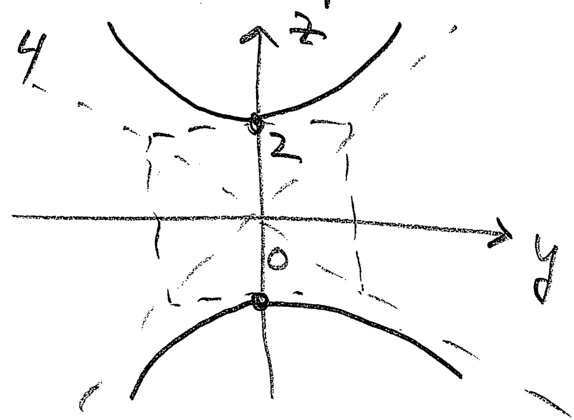
Level curves



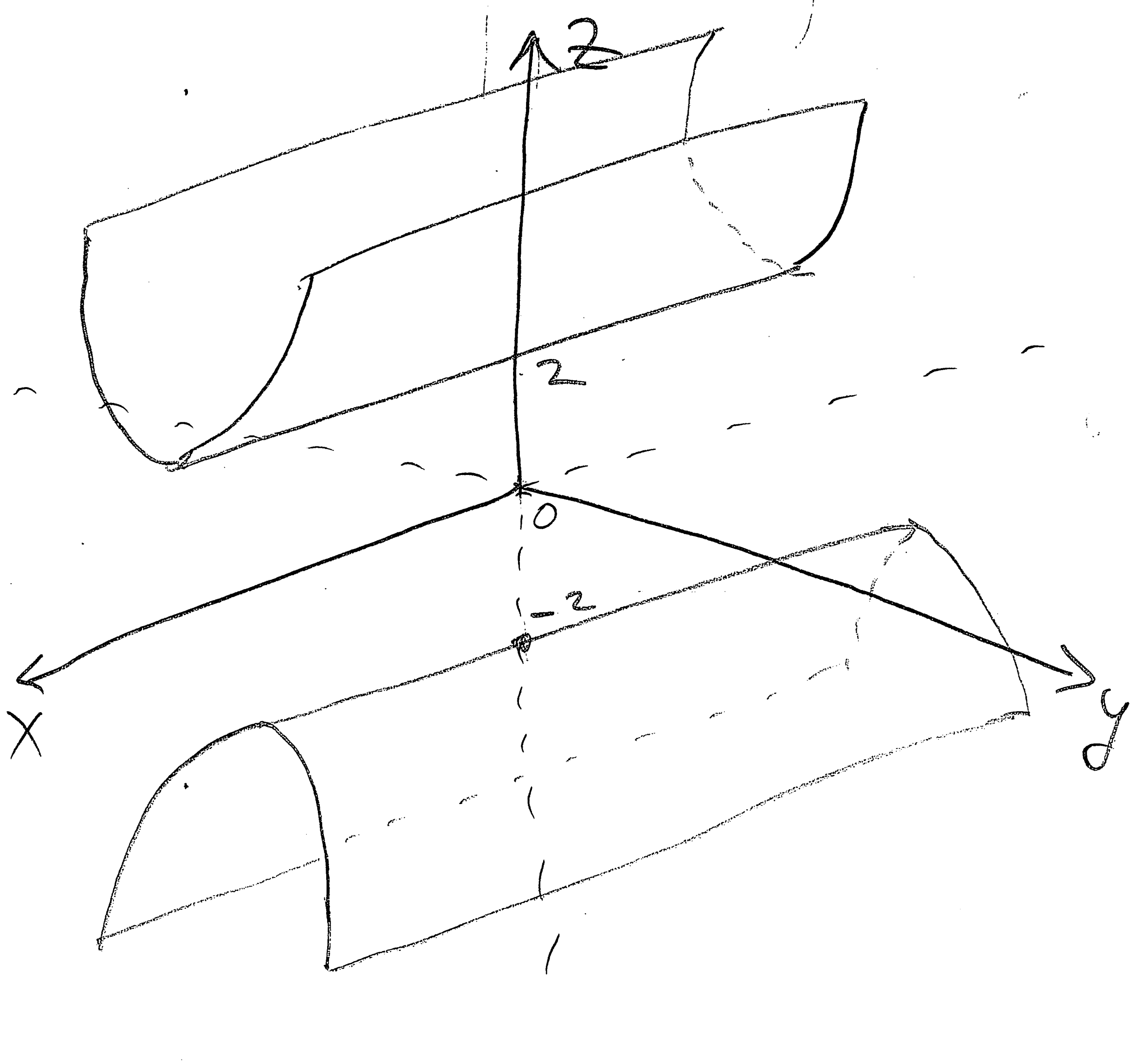
Graph



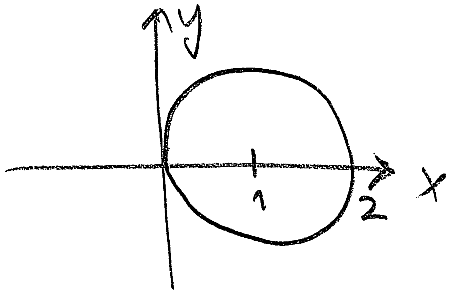
3 Page 87 #29: $z^2 = y^2 + 4$ is a hyperbola in the yz-plane: $z^2 - y^2 = 4$



However, in $\mathbb{R}^3$ there is no restriction on x and hence $\{x = k\} \cap$ graph is always the same hyperbola. This is a "generalized cylinder":



page 87 # 30 : $x^2+y^2-2x=0$, complete the square to get $(x-1)^2+y^2=1$. On xy plane this is a circle



But in $\mathbb{R}^3$, since there is no restriction on z, this is a cylinder: