Homework Set # 7

1. Maximize $f(x, y, z) = z - 2x + y$ subject to the two constraints $x^2 + y^2 = 8$ and $y - z = 1$.

2. Given the system of equations $\begin{cases} x + 2z + t^2 + u^2 + e^{2y} = 5 \\ x^2 + u + z + ty + y^3 = 0 \end{cases}$, is it possible to solve for the functions $u = u(x, z, t)$ and $y = y(x, z, t)$ in a neighborhood of $(x_0, y_0, z_0, t_0, u_0) = (0, 0, 1, 1, -1)$?

3. Consider the lamina (thin plate in $\mathbb{R}^2$) bounded below by $y = x^2 - 1$ and bounded above by $y = 3$. If the temperature at (x, y) is $T(x, y) = xy - x^2 + 10$, where is the coldest temperature on the plate? What is this coldest temperature?

4. **(§4.1)** **Page 227:** # 23.

5. Find the length of these curves:
   (a) $\mathbf{r}(t) = (2t)\,\mathbf{i} + t^2\mathbf{j} + \frac{1}{3}t^3\,\mathbf{k}, \quad 0 \leq t \leq 1.$
   (b) $y = \frac{1}{3}\,x^{\frac{3}{2}}$ from $(0, 0)$ to $(9, 9)$.

6. If $f, g, \phi : \mathbb{R} \longrightarrow \mathbb{R}$ are C^1 real-valued functions and $\mathbf{c}(t) = f(t)\,\mathbf{i} + g(t)\,\mathbf{j}$, prove that

$$\frac{d}{dt}\Big(\phi(t)\,\mathbf{c}(t)\Big) = \phi(t)\,\mathbf{c}'(t) + \phi'(t)\,\mathbf{c}(t)$$