Homework Set # 11 (PRACTICE)

1. Compute the line integral $\int_C \tilde{\mathbf{F}} \bullet d\tilde{\mathbf{s}}$, where $\tilde{\mathbf{F}} = (2xe^{2y} - \cos y + 2)\,\mathbf{i} + (2x^2e^{2y} + x\sin y)\,\mathbf{j}$ and C is any smooth curve starting at $(1,0)$ and ending at $(2,0)$. (Hint: Is $\tilde{\mathbf{F}}$ a gradient field ?)

2. Find the mass of a wire in the shape of a helix $\mathbf{c}(t) = (\cos 2t,\, \sin 2t,\, t)$ for $0 \le t \le \frac{\pi}{2}$, if the density is $f(x,y,z) = 16y$.

3. Compute the area of the surface $S\colon\ \Phi(u,v) = (u\cos v,\, u\sin v,\, v)$, where $D: 0 \le u \le \sqrt{8},\, 0 \le v \le u$.

4. Compute the surface integrals $\iint_S x\,dS$ and $\iint_S \tilde{\mathbf{F}} \bullet d\tilde{\mathbf{S}}$, where $\tilde{\mathbf{F}}(x,y,z) = (z,\, 4x,\, 2y+1)$ and S is that part of the plane $\frac{x}{2} + y + z = 1$ in the 1^{st} octant with upward normal.

5. Compute the flux of $\tilde{\mathbf{F}}$ across S, where $\tilde{\mathbf{F}}(x,y,z) = y\mathbf{i} - x\,\mathbf{j} + z\,\mathbf{k}$ and S is that part of the paraboloid $z = 9 - x^2 - y^2$ which lies above the plane $z = 5$ and $\tilde{\mathbf{N}}$ is the upward unit normal. What is the value of $\iint_S \left(\tilde{\mathbf{F}} \bullet \tilde{\mathbf{N}}\right) dS$?

6. Evaluate the line integral $\int_C x^2y\,dx + y^2\,dy$, along the simple closed curve C that is the positively oriented boundary of the region between $y = 4 - x^2$ and $y = 0$.

7. Compute the line integral $\int_C (-5xy)\;dy + (x^3 + \cos^2 x - 4y)\,dx$, where C is the positively oriented boundary of the rectangle $R = [0,\,2] \times [0,\,3]$.

8. Using **Green's Theorem** find the value of the line integral $\int_C y\,dx + (x^2 + y^2)\,dy$, where C is the circle $(x-3)^2 + y^2 = 9$ traversed in a positive direction:

Hint: Use fact that the centroid of the region bounded by C is $(\overline{x},\, \overline{y}) = (3,\,0)$

9. Let S be the surface $z = x^2 + y^2$ below $z = 4$ and downward normal $\mathbf{n}$ and $\tilde{\mathbf{F}}(x,y,z) = (-yz,\, xz,\, y)$. Compute $\iint_S \left(\nabla \times \tilde{\mathbf{F}}\right) \bullet d\tilde{\mathbf{S}}$, using **Stokes' Theorem**: $\iint_S \left(\nabla \times \tilde{\mathbf{F}}\right) \bullet d\tilde{\mathbf{S}} = \int_C \tilde{\mathbf{F}} \bullet d\tilde{\mathbf{s}}$.

10. Let $\tilde{\mathbf{F}} = x\,\mathbf{i} + y\,\mathbf{j} - 3z\,\mathbf{k}$. Compute $\iint_S \tilde{\mathbf{F}} \bullet d\tilde{\mathbf{S}}$, where S is the closed surface consisting of that part of the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 3$, including the top, with outward normal. Compute $\iint_S \tilde{\mathbf{F}} \bullet d\tilde{\mathbf{S}}$ directly by using the **Divergence Theorem**.

11. **(§8.3) Page 459:** # 1.