Summary of Path & Line Integrals and Surface Integrals

Path & Line Integrals	**Surface Integrals**
Path $\mathbf{c}(t) : I = [a,b] \to \mathbb{R}^n; \quad C = \mathbf{c}(I)$	Surface $\Phi(u,v) : D \subset \mathbb{R}^2 \to \mathbb{R}^3; \quad S = \Phi(D)$
$ds = \lVert\mathbf{c}'(t)\rVert\, dt =$ arc length differential	$dS = \lVert\Phi_u \times \Phi_v\rVert\, du\, dv =$ surface area differential
$\int_C ds = \int_a^b \lVert\mathbf{c}'(t)\rVert\, dt =$ Length of C	$\iint_S dS = \iint_D \lVert\Phi_u \times \Phi_v\rVert\, du\, dv =$ Surface area of S
$\int_C f\, ds = \int_a^b f(\mathbf{c}(t))\, \lVert\mathbf{c}'(t)\rVert\, dt$ (independent of orientation of C)	$\iint_S f\, dS = \iint_D f\big(\Phi(u,v)\big)\, \lVert\Phi_u \times \Phi_v\rVert\, du\, dv$ (independent of normal vector $\tilde{\mathbf{N}}$)
$d\,\tilde{\mathbf{s}} = \mathbf{c}'(t)\, dt$ Note: $\lVert d\,\tilde{\mathbf{s}}\rVert = ds$	$d\,\tilde{\mathbf{S}} = (\Phi_u \times \Phi_v)\; du\, dv$ Note: $\lVert d\,\tilde{\mathbf{S}}\rVert = dS$
$\int_C \tilde{\mathbf{F}} \bullet d\tilde{\mathbf{s}} = \int_a^b \tilde{\mathbf{F}}(\mathbf{c}(t)) \bullet \mathbf{c}'(t)\, dt$ (depends on orientation of C)	$\iint_S \tilde{\mathbf{F}} \bullet d\tilde{\mathbf{S}} = \iint_D \tilde{\mathbf{F}}\big(\Phi(u,v)\big) \bullet (\Phi_u \times \Phi_v)\; du\, dv$ (depends on normal vector $\tilde{\mathbf{N}}$)
$\int_C \tilde{\mathbf{F}} \bullet d\tilde{\mathbf{s}} = \int_C \left(\tilde{\mathbf{F}} \bullet \tilde{\mathbf{T}}\right)\, ds$ $=$ CIRCULATION of $\tilde{\mathbf{F}}$ around C, $\tilde{\mathbf{T}} = \dfrac{\mathbf{c}'(t)}{\lVert\mathbf{c}'(t)\rVert} =$ unit tangent vector to C	$\iint_S \tilde{\mathbf{F}} \bullet d\tilde{\mathbf{S}} = \iint_S \left(\tilde{\mathbf{F}} \bullet \tilde{\mathbf{N}}\right) dS$ $=$ FLUX of $\tilde{\mathbf{F}}$ across S in direction $\tilde{\mathbf{N}}$, $\tilde{\mathbf{N}} = \dfrac{\Phi_u \times \Phi_v}{\lVert\Phi_u \times \Phi_v\rVert} =$ unit normal vector to S