MATH 362 - PRACTICE PROBLEMS

1. Does $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2y}{x^4+2y^2}$ exist ? Why or why not ?

2. Let $h : \mathbb{R}^3 \to \mathbb{R}^2$ and $g : \mathbb{R}^2 \to \mathbb{R}^3$ be functions defined by

$$h(u,v,w) = (u^2,\, v^2 - 3w) \quad \text{and} \quad g(x,y) = (2x+1,\, x^2+y^2,\, y)\,.$$

Compute $D(h \circ g)$.

3. If $z = z(x,y)$ is defined implicitly by the equation $z + ye^z - x = 100$, compute $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial^2 z}{\partial x^2}$.

4. If $f : \mathbb{R}^2 \to \mathbb{R}$ is differentiable, show that $\nabla(f^3) = 3f^2\,\nabla(f)$.

5. Find the equation of the tangent plane to the surface $x^2+z^2 = ye^{3z}+2$ at the point $(1,-1,0)$.

6. If $u(x,t) = f(x+at)$ and $f \in C^2$ on $\mathbb{R}$, show that $a^2u_{xx} = u_{tt}$.

7. A particle travels along the path $\vec{\mathbf{c}}(t) = (2t^2, 3t+1)$. At $t = 1$, the particle flies off along a line tangent to the path with a speed of 2 units/sec. Where is the particle 5 seconds after it leaves the path ?

8. Given that $\vec{\mathbf{F}}(x,y,z) = (xz^4, e^{2x}, zy^2)$, compute curl $\vec{\mathbf{F}}$, $\nabla \cdot \vec{\mathbf{F}}$, and $\nabla(\operatorname{div}\vec{\mathbf{F}})$.

9. Find all critical points of $f(x,y) = x^2 + 6xy + 6y^3 + 1$ and determine if they are relative maxima, minima or saddle points.

10. A wire has shape of the ellipse $x^2 + 2y^2 = 9$ and the temperature of the wire at the point (x,y) is $T(x,y) = 2x - 8y + 12$. What is the hottest temperature on the wire and where does it occur ?

11. Compute these integrals:

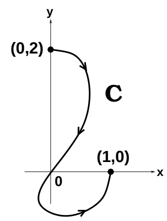
(a) $I = \displaystyle\iint_D 30\,x\,dA$, where D is the bounded region in the first quadrant between $y = x$ and $x = y^2$.

(b) $J = \displaystyle\int_0^2 \int_{\frac{y}{2}}^1 \frac{1}{(1+x^2)^2}\,dx\,dy$

12. Compute the path integral $\displaystyle\int_C \frac{(x+y^2+1)}{\sqrt{4y^2+1}}\,ds$, where C is the curve $x = 4 - y^2$, $-1 \le y \le 3$.

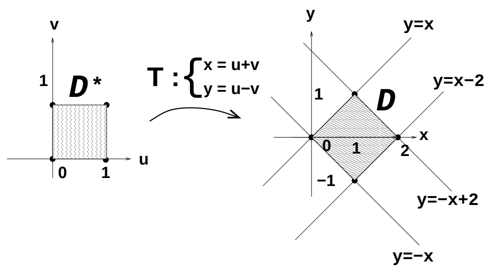
13. Find the length of the path $\vec{\mathbf{c}}(t) = (2t, t^2, \ln t)$ between the points $(2, 1, 0)$ and $(4, 4, \ln 2)$.

14. If $\vec{\mathbf{F}}(x, y) = (2x + y^2 e^x)\,\vec{\mathbf{i}} + (3 + 2ye^x)\,\vec{\mathbf{j}}$ and C is the curve shown below, compute the line integral $\displaystyle\int_C \vec{\mathbf{F}} \cdot d\,\vec{\mathbf{s}}$.



15. Given that the mapping $T(u, v) = (u + v, u - v)$ maps the region D^* one-to-one and onto the region D, shown below, use the Change of Variables Formula to compute the double integral

$$I = \iint_D 3\left(x^2 - y^2\right)\, dx\, dy\,.$$



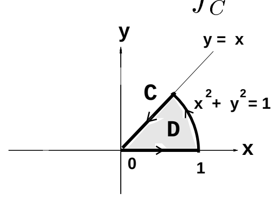
16. Let W be the "ice cream cone" shaped solid between the sphere $x^2 + y^2 + z^2 = 4$ and the cone $z = \sqrt{x^2 + y^2}$.

(a) Express the solid W in Cylindrical and Spherical Coordinates.

(b) Find the volume of the "ice cream cone" shaped solid W.

17. Sketch the solid W whose volume is given by $\displaystyle\int_0^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{0} \int_{x^2+y^2}^{2} dz\, dy\, dx.$

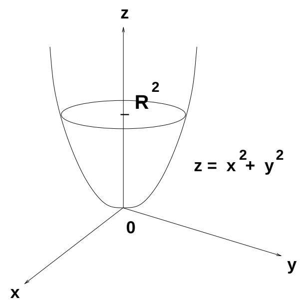
18. If C is the closed curve shown below, compute $\displaystyle\int_C (2 + \cos x)\, dx + (3x^2 + 3y^2)\, dy.$



19. Let S denote the oriented surface which is that part of the paraboloid $z = 3 - x^2 - y^2$ above the plane $z = 2$. If $\vec{\mathbf{F}}(x, y, z) = \dfrac{x}{2}\,\vec{\mathbf{i}} + \dfrac{y}{2}\,\vec{\mathbf{j}} + z\,\vec{\mathbf{k}}$, compute the flux of $\vec{\mathbf{F}}$ across S.

20. If S is that part of the paraboloid $z = x^2 + y^2$ which lies below the plane $z = R^2$ as shown below ($R > 0$ is a fixed constant), show that

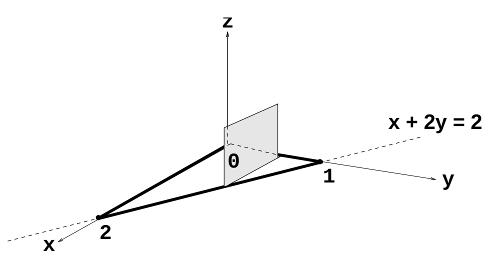
$$\iint_S \frac{1}{\sqrt{4z+1}}\, dS = \pi R^2 .$$



21. Evaluate $\displaystyle\iint_S (\nabla \times \vec{\mathbf{F}}) \cdot d\vec{\mathbf{S}}$, where S is the upper hemisphere $x^2 + y^2 + z^2 = 1$, $x \geq 0$ and $\vec{\mathbf{F}}(x, y, z) = x^3\vec{\mathbf{i}} - y^3\vec{\mathbf{j}}$.

22. Use Stokes' Theorem to compute $\displaystyle\int_C -y^3 dx + x^3 dy - z^3 dz$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$ and the orientation of C corresponds to counterclockwise motion in the xy plane.

23. The base of a solid S lies in the triangular region in the $xy-$plane between $x + 2y = 2$ and the coordinate axes as shown. If each plane section of S parallel to the $x-$axis is a square whose base lies in the triangular region, then the volume of S is



A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. 1

D. $\frac{4}{3}$

E. $\frac{5}{3}$

24. Given that $f'(u) = \dfrac{u}{1+u^3}$, if $z = f(u)$ and $u = x^2 + 2y$, then $\dfrac{\partial z}{\partial x} =$

A. $\dfrac{2x}{1+(x^2+2y)^3}$

B. $\dfrac{4x^2+2x}{1+(x^2+2y)^3}$

C. $\dfrac{x^2+2y}{1+(x^2+2y)^3}$

D. $\dfrac{2x^3+4xy}{1+(x^2+2y)^3}$

E. $\dfrac{2x(1-2(x^2+2y)^3)}{(1+(x^2+2y)^3)^2}$

25. If D is the region $1 \leq x^2 + y^2 \leq 4$, in the first quadrant, then $\displaystyle\iint_D 2\ln(x^2+y^2)\,dx\,dy =$

A. $\dfrac{\pi}{2}(8\ln 2 - 3)$

B. $\dfrac{\pi}{2}(\ln 8 - 3)$

C. $\dfrac{\pi}{2}(4\ln 2 - 3)$

D. $\dfrac{\pi}{2}(4\ln 2 - 3)$

E. $\dfrac{3\pi\ln 2}{4}$