10/07/16

MA 362 Challenge Problems

1 Distance Between Two Planes

Show that the distance d between the two parallel planes

$$Ax + By + Cz + D_1 = 0 \quad \text{and} \quad Ax + By + Cz + D_2 = 0$$

is given by $d = \dfrac{|D_1 - D_2|}{\sqrt{A^2 + B^2 + C^2}}$.

2 Mixed Partials Not Always Equal

Let $f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{(x^2 + y^2)} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$.

Using the definition of partial derivative (using limits), show that

a $\quad \dfrac{\partial f}{\partial x}(0, 0) = 0$ and $\dfrac{\partial f}{\partial y}(0, 0) = 0$

b $\quad \dfrac{\partial^2 f}{\partial x \, \partial y}(0, 0) = 1$ and $\dfrac{\partial^2 f}{\partial y \, \partial x}(0, 0) = -1$

(Thus $f_{yx} \neq f_{xy}$ for this special function f.)

3 W.L.O.G. Quadratic Forms $Q(\mathbf{x}) = \mathbf{x}A\mathbf{x}^T$ have matrix A symmetric

If $A = [a_{ij}]$ is an $n \times n$ matrix and if $B = [b_{ij}]$, where $b_{ij} = \frac{1}{2}(a_{ij} + a_{ji})$, prove that for all $\mathbf{x} \in \mathbb{R}^n$,

$$\mathbf{x}\, A\, \mathbf{x}^T = \mathbf{x}\, B\, \mathbf{x}^T \, .$$

4 _

5 _

6 _