Undetermined Coefficients

Method to find a particular solution $y_p(t)$ to a linear nonhomogeneous equation with constant coefficients $L[y] = g(t)$, where $g(t)$ has a **very, very** special form.

2^{nd} Order Nonhomogeneous Equations: $\boxed{L[y] = ay'' + by' + cy = g(t)}$

Factor its Characteristic Polynomial $P(r) = ar^2 + br + c$. The roots and their multiplicities are used below:

$g(t)$	Form of $y_p(t)$
$P_m(t) = \left(b_0 + b_1 t + \cdots + b_m t^m\right)$	$t^s\left\{A_0 + A_1 t + \cdots + A_m t^m\right\}$ $s =$ multiplicity of $\boxed{\boldsymbol{r = 0}}$ in $P(r)$
$e^{\alpha t} P_m(t)$	$t^s\left\{e^{\alpha t}\left[A_0 + A_1 t + \cdots + A_m t^m\right]\right\}$ $s =$ multiplicity of $\boxed{\boldsymbol{r = \alpha}}$ in $P(r)$
$e^{\alpha t} P_m(t) \begin{cases} \cos \beta t \\ \textbf{or} \\ \sin \beta t \end{cases}$	$t^s\left\{e^{\alpha t}\left[(A_0 + A_1 t + \cdots + A_m t^m)\cos \beta t + (B_0 + B_1 t + \cdots + B_m t^m)\sin \beta t\right]\right\}$ $s =$ multiplicity of $\boxed{\boldsymbol{r = \alpha + \beta i}}$ in $P(r)$

Equivalently, s is the *smallest* integer $(0, 1,$ or $2)$ such that no term in $y_p(t)$ is a solution to $L[y] = 0$.

n^{th} Order Nonhomogeneous Equations: $\boxed{L[y] = a_0 y^{(n)} + a_1 y^{(n-1)} + \cdots + a_{n-1} y' + a_n y = g(t)}$

Factor its Characteristic Polynomial $P(r) = a_0 r^n + a_1 r^{n-1} + \cdots + a_{n-1} r + a_n$. The roots and their multiplicities are used below:

$g(t)$	Form of $y_p(t)$
$P_m(t) = \left(b_0 + b_1 t + \cdots + b_m t^m\right)$	$t^s\left\{A_0 + A_1 t + \cdots + A_m t^m\right\}$ $s =$ multiplicity of $\boxed{\boldsymbol{r = 0}}$ in $P(r)$
$e^{\alpha t} P_m(t)$	$t^s\left\{e^{\alpha t}\left[A_0 + A_1 t + \cdots + A_m t^m\right]\right\}$ $s =$ multiplicity of $\boxed{\boldsymbol{r = \alpha}}$ in $P(r)$
$e^{\alpha t} P_m(t) \begin{cases} \cos \beta t \\ \textbf{or} \\ \sin \beta t \end{cases}$	$t^s\left\{e^{\alpha t}\left[(A_0 + A_1 t + \cdots + A_m t^m)\cos \beta t + (B_0 + B_1 t + \cdots + B_m t^m)\sin \beta t\right]\right\}$ $s =$ multiplicity of $\boxed{\boldsymbol{r = \alpha + \beta i}}$ in $P(r)$

Equivalently, s is the *smallest* integer $(0, 1, 2 \cdots,$ or $n)$ such that no term in $y_p(t)$ is a solution to $L[y] = 0$.