Problem Set # 12 (Optional)

(due: April 30)

1. **Page 178** : # 37(b).

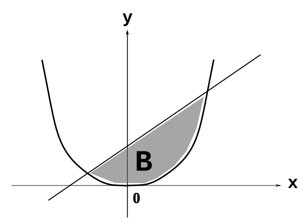
2. **Page 243** : # 5.

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ and $g : \mathbb{R}^2 \to \mathbb{R}$ be differentiable, prove that $\nabla(fg) = f\nabla g + g\nabla f$.

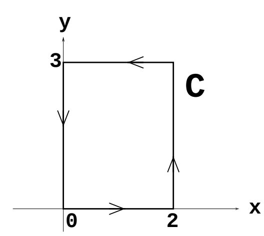
4. If $g(x, y, z) = (x,\ x + y,\ x^2 + z,\ z)$ and $f(x_1, x_2, x_3, x_4) = (x_1^3 + x_3, x_2^2 - x_4)$, then compute $D(f \circ g)(1, 1, 1)$.

5. If $z = z(x, y)$ is defined implicitly by the equation $\boldsymbol{x^2 + z^3 + 3xy - 3z = 1}$, compute $\dfrac{\partial z}{\partial y}$.

6. The base of a solid S lies in the region B between $y = x^2$ and $y = x + 2$ in the xy-plane as shown below. Plane sections perpendicular to the x-axis are squares with one side in the xy-plane. Find the volume of S.



7. Compute $\displaystyle\int_C -5xy\,dy + (x^3 + \cos^2 x - 4y)\,dx$ where C is as shown:



8. If $\vec{\mathbf{F}}(x, y, z) = -yz\,\vec{\mathbf{i}} + xz\,\vec{\mathbf{j}} + y\,\vec{\mathbf{k}}$ and S is that part of $z = x^2 + y^2$ below $z = 4$ (the normal $\vec{\mathbf{n}}$ points downward), compute $\displaystyle\iint_S (\nabla \times \vec{\mathbf{F}}) \cdot d\vec{\mathbf{S}}$ using Stokes' Theorem.

9. Let $\vec{\mathbf{F}}(x, y, z) = (x, y, -3z)$.

(a) Compute $\displaystyle\iint_{S_1} \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$ where S_1 is that part of $z = \sqrt{x^2 + y^2}$ below the plane $z = 3$ and $\vec{\mathbf{n}}$ is downward.

(b) Compute $\displaystyle\iint_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$, where S is the *closed* surface consisting of that part of $z = \sqrt{x^2 + y^2}$ below the plane $z = 3$ together with the top and $\vec{\mathbf{n}}$ points outward.

10. **Page 574** : # 10.