Comparison of Path & Line Integrals to Surface Integrals

Path & Line Integrals	Surface Integrals
Curve $C : \vec{\mathbf{c}}(t)$, where $a \le t \le b$ $\vec{\mathbf{c}}(t) = (x(t), y(t), z(t))$ $\vec{\mathbf{c}}\,'(t) = (x'(t), y'(t), z'(t))$	*Surface* $S : \Phi(u,v)$, where $(u,v) \in D$ $\Phi(u,v) = (x(u,v), y(u,v), z(u,v))$ $\vec{\mathbf{T}}_u = \left(\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u}\right) \quad \vec{\mathbf{T}}_v = \left(\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v}\right)$
$ds = \lvert\lvert \vec{\mathbf{c}}\,'(t) \rvert\rvert \, dt =$ differential of arc length	$dS = \lvert\lvert \vec{\mathbf{T}}_u \times \vec{\mathbf{T}}_v \rvert\rvert \, dA =$ differential of surface area
$\int_C ds =$ length of C	$\iint_S dS =$ surface area of S
$\int_C f(x,y,z)\, ds = \int_a^b f(\vec{\mathbf{c}}(t))\, \lvert\lvert \vec{\mathbf{c}}\,'(t) \rvert\rvert \, dt$ (independent of orientation of C)	$\iint_S f(x,y,z)\, dS = \iint_D f(\Phi(u,v))\, \lvert\lvert \vec{\mathbf{T}}_u \times \vec{\mathbf{T}}_v \rvert\rvert \, du\, dv$ (independent of unit normal vector $\vec{\mathbf{n}}$)
$d\vec{\mathbf{s}} = \vec{\mathbf{c}}\,'(t)\, dt$	$d\vec{\mathbf{S}} = \left(\vec{\mathbf{T}}_u \times \vec{\mathbf{T}}_v\right) dA$
$\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = \int_a^b \vec{\mathbf{F}}(\vec{\mathbf{c}}(t)) \cdot \vec{\mathbf{c}}\,'(t)\, dt$ (depends on orientation of C)	$\iint_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}} = \iint_D \vec{\mathbf{F}}(\Phi(u,v)) \cdot \left(\vec{\mathbf{T}}_u \times \vec{\mathbf{T}}_v\right) du\, dv$ (depends on unit normal vector $\vec{\mathbf{n}}$)
$\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = \int_C \left(\vec{\mathbf{F}} \cdot \vec{\mathbf{T}}\right) ds$ The *circulation* of $\vec{\mathbf{F}}$ around C	$\iint_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}} = \iint_S \left(\vec{\mathbf{F}} \cdot \vec{\mathbf{n}}\right) dS$ The *flux* of $\vec{\mathbf{F}}$ across S in direction $\vec{\mathbf{n}} \perp S$