TOPICS - MIDTERM EXAM #1

1. Equations of lines and planes; dot or inner products; cross products (only in $\mathbb{R}^3$); properties of dot and cross products; cylindrical and spherical coordinates.

2. Level curves, level surfaces; sketching surfaces using level curves.

3. Definition of limit; computing limits using the **Limit Theorem**, **Squeeze Theorem** or **Continuity of Composition Theorem**; showing a limit does not exist; continuous functions.

4. Partial derivatives; gradient of $f : \mathbb{R}^n \to \mathbb{R}$; tangent planes to surfaces; (linear) approximation formula.

5. Derivative of a function $f : \mathbb{R}^n \to \mathbb{R}^m$ (if $f(\vec{\mathbf{x}}) = (f_1(\vec{\mathbf{x}}), f_2(\vec{\mathbf{x}}), \cdots, f_m(\vec{\mathbf{x}}))$, then $Df(\vec{\mathbf{x}}_0) = \left(\frac{\partial f_i}{\partial x_j}(\vec{\mathbf{x}}_0)\right)_{m\times n}$ is also called the $m \times n$ matrix of partial derivatives); differentiability; properties of derivatives.

6. Paths $\vec{\mathbf{c}}(t)$; velocity, speed, tangent vector; acceleration; arc length; reparameterizing by arc length; differentiation rules for $\vec{\mathbf{c}}(t)$.

7. **CHAIN RULE**; tree diagrams; implicit differentiation.

8. Directional derivative $D_{\vec{\mathbf{u}}}f(\vec{\mathbf{x}}) = \nabla f(\vec{\mathbf{x}}) \cdot \vec{\mathbf{u}}$ ($\vec{\mathbf{u}}$ must be a unit vector); basic properties of directional derivatives; rate of change of f along a path $\vec{\mathbf{c}}(t)$; ∇f is perpendicular (normal) to corresponding level set of f, i.e.,$\nabla f(\vec{\mathbf{x}}) \perp \{f(\vec{\mathbf{x}}) = C\}$.

9. Iterated and mixed partial derivatives; Hessian of a function $f : \mathbb{R}^n \to \mathbb{R}$ given by $Hf(\vec{\mathbf{x}}_0)(\vec{\mathbf{h}}) = \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{\partial^2 f_i}{\partial x_j^2}(\vec{\mathbf{x}}_0)h_i h_j$; **Taylor's Formula**, $\mathbf{1^{st}}$ and $\mathbf{2^{nd}}$ **order**.

10. Critical points of $f : \mathbb{R}^n \to \mathbb{R}$; local extrema using the 2^{nd} Partials Test or using basic principles; extremal problems over closed and bounded regions; constrained extremal problems; **Lagrange Multiplier Method**.