Topics - Midterm Exam #2

1. Paths $\vec{\mathbf{c}}(t)$; velocity, speed, tangent vector; acceleration; arc length; reparameterizing by arc length; differentiation rules for $\vec{\mathbf{c}}(t)$ (Page 262).

2. Vector fields $\vec{\mathbf{F}} : \mathbb{R}^n \to \mathbb{R}^n$; flow lines of a vector field $\vec{\mathbf{F}}$; *divergence* of $\vec{\mathbf{F}}$ (div $\vec{\mathbf{F}} = \nabla \cdot \vec{\mathbf{F}}$); *curl* of $\vec{\mathbf{F}}$ (curl $\vec{\mathbf{F}} = \nabla \times \vec{\mathbf{F}}$); basic properties of the del operator ∇ (Page 306); scalar curl of $\vec{\mathbf{F}} : \mathbb{R}^2 \to \mathbb{R}^2$; gradient fields $\vec{\mathbf{F}} = \nabla \psi$; potential functions.

3. Double integrals; Volumes by slices (*Cavalieri's Principle*); properites of double integrals; elementary regions in $\mathbb{R}^2$ (x-simple, y-simple); simple regions; *Fubini's Theorem*; iterated integrals; computing areas and volumes using double integrals.

4. Changing the order of integration in double integrals; *Mean Value Theorem for Double Integrals.*

5. Triple integrals; changing the order of integration in triple integrals; elementary regions in $\mathbb{R}^3$.

6. Jacobian determinants; **Change of Variables Formula in** $\mathbb{R}^2$:

If $T(u,v) = (x(u,v), y(u,v))$ then

$$\iint_D f(x,y)\,dx\,dy = \iint_{D^*} f(x(u,v), y(u,v)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\,dv\,.$$

7. Change of Variables Formula for *Polar Coordinates, Cylindrical* and *Spherical Coordinates.*

8. Applications of double and triple integrals: average values, center of mass, centroid.

9. **Path integrals**: $\displaystyle\int_C f\,ds = \int_a^b f(\vec{\mathbf{c}}(t))\,||\vec{\mathbf{c}}\,'(t)||\,dt$ (does not depend on orientation); applications.

10. **Line integrals**: $\displaystyle\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = \int_a^b \vec{\mathbf{F}}(\vec{\mathbf{c}}(t)) \cdot \vec{\mathbf{c}}\,'(t)\,dt$ (depends on orientation); line integrals of the form $\displaystyle\int_C P\,dx + Q\,dy + R\,dz$; **Work** $= \displaystyle\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$; reparameterization of paths and effect on path and line integrals.

11. **Fundamental Theorem of Calculus for Line Integrals**:

$$\int_{\vec{\mathbf{c}}} \nabla f \cdot d\vec{\mathbf{s}} = f(\vec{\mathbf{c}}(b)) - f(\vec{\mathbf{c}}(a))\,.$$

12. Parameterized surfaces $\Phi(u,v) = (x(u,v),\, y(u,v),\, z(u,v))$, $(u,v) \in D$; normal vector to a surface is $\vec{\mathbf{n}} = \vec{\mathbf{T}}_u \times \vec{\mathbf{T}}_v$, where $\vec{\mathbf{T}}_u = \left(\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u} \right)$ and $\vec{\mathbf{T}}_v = \left(\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v} \right)$; equation of tangent plane to a surface; area of a surface $A(S) = \displaystyle\iint_D ||\vec{\mathbf{T}}_u \times \vec{\mathbf{T}}_v||\,du\,dv.$

13. **Surface Integrals** of functions f and vector fields $\vec{\mathbf{F}}$:

$$\iint_S f\,dS = \iint_D f(\Phi(u,v))\,||\vec{\mathbf{T}}_u \times \vec{\mathbf{T}}_v||\,du\,dv$$

$$\iint_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}} = \iint_D \vec{\mathbf{F}}(\Phi(u,v)) \cdot (\vec{\mathbf{T}}_u \times \vec{\mathbf{T}}_v)\,du\,dv\,.$$