Worksheet # 1


2. The sum of the present value of 1 paid at the end of $n$ periods and 1 paid at the end of $2n$ periods is 1. Calculate $(1 + i)^{2n}$.

3. The amount of interest earned on $M$ for one year is $336$, while the equivalent amount of discount is $300$. Find $M$.

4. An investor puts $100$ into Fund X and puts $100$ into Fund Y. The Fund Y earns compound interest at the annual rate of $j$ ($j > 0$) and Fund X will earn simple interest at the annual rate of $1.05j$. At the end of 2 years both funds have the same amount. What is the amount in Fund Y at the end of 5 years? *(SHOW WORK)*
   
   (A) $150$ (B) $153$ (C) $157$ (D) $161$ (E) $165$

5. You may have already won $1,000,000$ from Publisher’s Y2K Clearing House. If you win, you will be paid in installments of $100,000$ immediately, $200,000$ a year from now, $300,000$ two years from now and $400,000$ three years from now. If you win and the interest rate is 12% annual effective, what is the value of your prize today? What is the value of the prize at the end of the third year assuming no withdrawals are made?

6. If a fund has a force of interest $\delta_t = \frac{1}{1 + 0.05t}$ and after 2 years it has grown to $3000$, how much was originally invested? *(SHOW WORK)*
   
   (A) 406.00 (B) 445.93 (C) 1156.63 (D) 2714.51 (E) 2727.27

7. If the amount of money in an account is $A(t) = 2000(1 + 0.002t^3 + 0.08t)$, find the force of interest $\delta_t$. What is $\delta_2$?

8. The force of interest of a fund is $\delta_t = \frac{6t}{t^2 + 1}$. Find the accumulation function $a(t)$.

9. Peter deposits 400 into a bank at time $t = 0$. During the first year, the bank credits interest at a nominal rate of 10% compounded semiannually. Peter makes an additional deposit of 42 into his bank at time $t = 1$. During the second year, the bank credits interest at a force of interest $\delta_t = \frac{1}{K + t}$. The total amount in Peter’s account at time $t = 2$ is 552. Calculate $K$. *(SHOW WORK)*
   
   (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

10. Fund A accumulates at a rate of 12% converted monthly. Fund B accumulates with a force of interest $\delta_t = \frac{t}{6}$, for all $t$. At time $t = 0$, 1 is deposited in each fund. $T$ is the time that the two funds are equal, $T > 0$. Determine $T$. *(SHOW WORK)*
    
    (A) $12 \ln 1.01$ (B) $12(\ln 1.12 - \ln 1.01)$ (C) $12 \ln 1.12$ (D) $144 \ln 1.01$ (E) $144 \ln 1.12$
11. The present value of a payment of 1004 at the end of $T$ months is equal to the present value of 314 after 1 month, 271 after 18 months and 419 after 24 months. The effective annual interest rate is 5%. Calculate $T$. (SHOW WORK)  
(A) 14  (B) 15  (C) 16  (D) 17  (E) 18

12. John borrows 1000 from Jane at an annual effective rate of interest $i$. He agrees to pay back 1000 after 6 years and 1366.87 after another 6 years. Three years after his 1$^{st}$ payment John repays the outstanding balance. What is the amount of John’s 2$^{nd}$ payment? (SHOW WORK)  
(A) 1020  (B) 1027  (C) 1048  (D) 1073  (E) 1094

13. You are given a loan on which interest is charged over a 4-year period as follows: 

(i) An effective rate of discount of 6% for the 1$^{st}$ year 
(ii) A nominal rate of discount of 5% compounded every 2 years for the 2$^{nd}$ year 
(iii) A nominal rate of interest of 5% compounded semiannually for the 3$^{rd}$ year 
(iv) A force of interest of 5% for the 4$^{th}$ year 

Calculate the annual effective rate of interest over the 4-year period. (SHOW WORK)  
(A) .0500  (B) .0525  (C) .0550  (D) .0575  (E) .0600

14. The present value of two payments of $100 each to be made at the end of $n$ years and $2n$ years is $100. If $i = 0.08$, find $n$. 

15. On 1/1/97, Kelly deposits $X$ into a bank account. The account is credited with simple interest at the rate of 10% per year. On the same date, Tara deposits $X$ into a different bank account. The account is credited interest using a force of interest $\delta_t = \frac{2t}{(t^2 + k)}$. From the end of the 4$^{th}$ year until the end of the 8$^{th}$ year, both accounts earn the same dollar amount of interest. Calculate $k$. 

16. You are given that Fund X accumulates at an interest rate of 8% compounded quarterly, and Fund Y accumulates at an interest rate of 6% compounded semiannually. At the end of 10 years, the total amount in the two funds combined is 1000. At the end of 5 years, the amount in Fund X is twice that in Fund Y. Calculate the total amount in the two funds at the end of 2 years. (SHOW WORK)  
(A) 560  (B) 570  (C) 580  (D) 590  (E) 600

17. Find the value of the following set of payments at time $t = 0$, $t = 3$, and at $t = 7$, assuming an annual effective rate of 6%:

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(A) 1020  (B) 1027  (C) 1048  (D) 1073  (E) 1094
Answers

1. (1-8) : 700 ; (1-9) : 14,710.39 ; (1-10) : i = 13% ; (1-19) : \( L = 2700 \);
(1-20) : Account C is most advantageous to the investor (equivalent annual rate is highest);
Account A is most advantageous to Acme Trust since equivalent annual rate is smallest ;
(1-21) : 3947.80 ; (1-22) : \( i = 0.1236, d^{(4)} = 0.1149, i^{(12)} = 0.1171, j = 0.00976 \);
(1-23) : \( j = 0.07, i^{(2)} = 0.14, i = 0.1449, d^{(3)} = 0.1323 \) ; (1-24) : \( n = 24 \)

2. \( \frac{4}{6-2\sqrt{4}} \)

3. \( M = 2800 \quad 4. \quad D \quad 5. \quad PV = $802, 442 ; \quad FV = $1, 127, 373 \)

6. \( B \quad 7. \quad \delta_t = \frac{0.006t^2 + 0.08}{1 + 0.002t + 0.08t} \), \( \delta_2 = 0.0884 \)

8. \( a(t) = (t^2 + 1)^3 \)

9. \( B \quad 10. \quad D \)

11. \( B \quad 12. \quad B \quad 13. \quad C \quad 14. \quad n = 6.25 \quad 15. \quad k = 120 \quad 16. \quad A \)

17. \( @ t = 0 \) Value = 521.24 ; \( @ t = 3 \) Value = 620.80 ; \( @ t = 7 \) : Value = 783.75