GREEN EXAM

1. Which of the following three statements are **TRUE** ?

(I) $2^x\,5^{2x} = 50^x$ (II) $e^{a+2b} = e^a + e^{2b}$ (III) $\ln\sqrt{\frac{a^3}{e^2}} = \frac{3}{2}\ln a - 1$

$$2^x 5^{2x} = 2^x (5^2)^x = 2^x (25)^x$$

$$= (2(25))^x = 50^x \quad \text{(I) True}$$

$$e^{a+2b} = e^a \cdot e^{2b} \quad \text{(II) False}$$

A. Only (I)

[✓ **B.** Only (I) and (III)]

C. Only (I) and (II)

D. Only (II) and (III)

E. Only (III)

$$\ln\sqrt{\frac{a^3}{e^2}} = \frac{1}{2}\ln\left(\frac{a^3}{e^2}\right) = \frac{1}{2}\left\{\ln(a^3) - \ln(e^2)\right\}$$

$$= \frac{1}{2}\left\{3\ln a - 2\ln e\right\} = \frac{1}{2}\left\{3\ln a - 2\right\} \quad \text{(III) True}$$

2. Find $f^{-1}(7)$ if $f(x) = 1 + 2e^{x-3}$.

Soln 1 - $y = 1 + 2e^{x-3} \Rightarrow \frac{y-1}{2} = e^{x-3}$

$$\Rightarrow \ln\left(\frac{y-1}{2}\right) = x - 3 \Rightarrow \underbrace{3 + \ln\left(\frac{y-1}{2}\right)}_{f^{-1}(y)} = x$$

A. $f^{-1}(7) = 3 + \ln 6$

B. $f^{-1}(7) = 3 - \ln 3$

C. $f^{-1}(7) = -1 + \ln 2$

D. $f^{-1}(7) = 3 + \ln 2$

[✓ **E.** $f^{-1}(7) = 3 + \ln 3$]

$\therefore f^{-1}(x) = 3 + \ln\left(\frac{x-1}{2}\right)$ so $f^{-1}(7) = 3 + \ln 3$

Soln 2 - If you want the value $x = f^{-1}(7)$, then $f(x) = 7$

Hence just solve for x when $1 + 2e^{x-3} = 7$

$$\Rightarrow 2e^{x-3} = 6 \Rightarrow e^{x-3} = 3 \Rightarrow (x-3) = \ln 3$$

$$\Rightarrow x = 3 + \ln 3$$

3. If the graph of $f(x)$ is shifted 2 units to the left, reflected across the $y-$axis, and finally shifted up 3 units, the resulting function is

$f(x)$

$f(x+2)$, 2 units left

$f(-x+2)$, reflected across y-axis

$f(-x+2)+3$, shifted up 3 units

$h(x) = f(-x+2)+3$

A. $h(x) = -f(x+2) - 3$

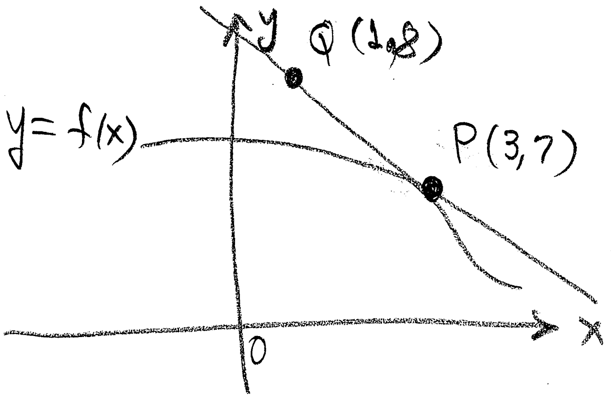
[✓ **B.**] $h(x) = f(-x+2) + 3$

C. $h(x) = f(x-2) - 3$

D. $h(x) = -f(x+2) + 3$

E. $h(x) = -f(-x-2) + 3$

4. If the tangent line to the curve $y = f(x)$ at $P(3,7)$ passes through the point $Q(1,8)$, find $f'(3)$.



A. $f'(3) = \frac{2}{3}$

[✓ **B.**] $f'(3) = -\frac{1}{2}$

C. $f'(3) = \frac{7}{3}$

D. $f'(3) = -\frac{1}{3}$

E. $f'(3) = 1$

Slope of tangent line = slope of line through P, Q

$$= \frac{8-7}{1-3} = -\frac{1}{2}$$

$$\overset{\overset{0}{\uparrow \text{ as } x\to 0}}{-x^2} \le x^2\cos\left(\tfrac{1}{\pi x}\right) \le \overset{\overset{0}{\uparrow \text{ as } x\to 0}}{x^2}$$

$\searrow 0$ as $x \to 0$

Squeeze Thm

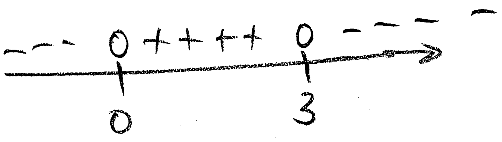
5. Which of these three statements are **TRUE** ?

FALSE → (I) $\lim_{x\to 0}\left\{x^2\cos\left(\frac{1}{\pi x}\right)\right\}$ DNE (does not exist).

TRUE (II) The function $f(x) = \ln\left(3x - x^2\right)$ is continuous for $0 < x < 3$.

(III) $\lim_{x\to\infty} \frac{x}{\sqrt{5x^3-2}} = 0.$ TRUE

$3x - x^2 > 0 \Leftrightarrow$



A. Only (I)

B. Only (I) and (III)

C. Only (I) and (II)

[✓ **D.**] Only (II) and (III)

E. Only (III)

$$\lim_{x\to\infty}\frac{x}{\sqrt{5x^3-2}} = \lim_{x\to\infty}\frac{x}{\sqrt{x^3\left(5-\frac{2}{x^3}\right)}}$$

$$= \lim_{x\to\infty}\frac{x}{x^{3/2}\sqrt{5-\frac{2}{x^3}}} = \lim_{x\to\infty}\frac{1}{x^{1/2}\sqrt{5-\frac{2}{x^3}}} = 0$$

6. $\lim_{x\to 1}\left(\frac{1-x}{1-\sqrt{x}}\right)^2 =$ $\left(\lim_{x\to 1}\frac{1-x}{1-\sqrt{x}}\right)^2$

[✓ **A.**] 4

B. 1

C. $\frac{1}{2}$

D. $\frac{1}{\sqrt{2}}$

E. DNE

$$= \left(\lim_{x\to 1}\frac{1-x}{1-\sqrt{x}}\cdot\frac{1+\sqrt{x}}{1+\sqrt{x}}\right)^2$$

$$= \left(\lim_{x\to 1}\frac{(1-x)(1+\sqrt{x})}{1-x}\right)^2 = \left(\lim_{x\to 1} 1+\sqrt{x}\right)^2 = 2^2 = 4$$

7. Find a and b so the function $f(x)$ defined below is continuous for all values of x .

$X=1,\ X=2$

only places we need to check.

$$f(x)=\begin{cases} x^2, & x<1 \\ ax+b, & 1\le x\le 2 \\ \dfrac{x^2+1}{x-1}, & x>2 \end{cases}$$

① $\lim\limits_{X\to 1^-} f(X) = \lim\limits_{X\to 1^+} f(x) \Rightarrow \lim\limits_{X\to 1^-} X^2 = \lim\limits_{X\to 1^+} (ax+b)$

i.e. $1 = a+b$

② $\lim\limits_{X\to 2^-} f(x) = \lim\limits_{X\to 2^+} f(x) \Rightarrow \lim\limits_{X\to 2^-} (ax+b) = \lim\limits_{X\to 2} \dfrac{x^2+1}{x-1}$

$\Rightarrow\ 2a+b=5 \qquad \therefore \begin{cases} 1=a+b \\ 5=2a+b \end{cases} \Rightarrow\ a=4,\ a=-3$

A. $a=1,\ b=1$

B. $a=2,\ b=-1$

[✓ **C.**] $a=4,\ b=-3$

D. $a=1,\ b=5$

E. No values of a or b, since $\dfrac{x^2+1}{x-1}$ is not continuous at $x=1$

8. Evaluate this limit: $\lim\limits_{x\to\infty}\left(\sqrt{x^2+3x}-x\right)$.

$$\sqrt{x^2+3x}-x = \left(\sqrt{x^2+3x}-x\right)\left(\frac{\sqrt{x^2+3x}+x}{\sqrt{x^2+3x}+x}\right)$$

$$= \frac{(x^2+3x)-x^2}{\sqrt{x^2+3x}+x} = \frac{3x}{\sqrt{x^2+3x}+x}$$

$$= \frac{3x}{\sqrt{x^2\left(1+\frac{3}{x}\right)}+x} = \frac{3x}{|x|\sqrt{1+\frac{3}{x}}+x} = \frac{3x}{x\sqrt{1+\frac{3}{x}}+x}$$

$$= \frac{3}{\sqrt{1+\frac{3}{x}}+1} \rightarrow \frac{3}{1+1} = \frac{3}{2}$$

as $x\to\infty$

A. $\frac{\sqrt{3}}{2}$

[✓ **B.**] $\frac{3}{2}$

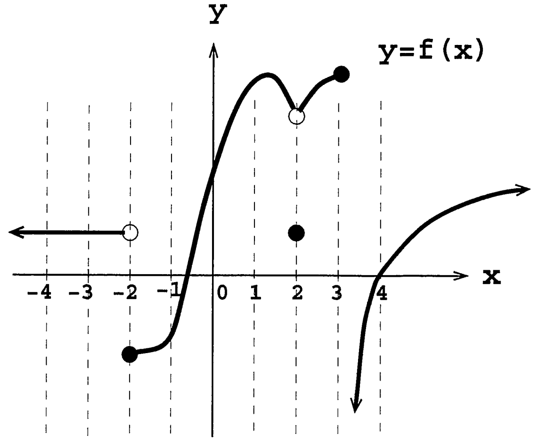
C. 0

D. 3

E. DNE

9. Which of the following statements are **TRUE** for this function?

$\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$



(I) $\lim_{x\to 3} f(x)$ exists FALSE

(II) f is continuous on $[-2, 2)$ TRUE

(III) $\underbrace{\lim_{x\to -2^-} f(x)}_{>0} = \underbrace{f(-2)}_{<0}$ FALSE

A. Only (I)

B. Only (I) and (III)

C. Only (I) and (II)

D. Only (II) and (III)

[✓ **E.**] Only (II)

10. Find all vertical and horizontal asymptotes of the function $f(x) = \dfrac{(x-1)^3(x-3)^2}{x^2(x^2-9)}$.

	Vertical Asymptotes		**Horizontal Asymptotes**
A.	$x=-3,\ x=0,\ x=3$	and	$y=0$
B.	$x=-3,\ x=0$	and	$y=1$
[✓ **C.**]	$x=-3,\ x=0$	and	none
D.	$x=0,\ x=3$	and	$y=1$
E.	$x=0,\ x=3$	and	$y=-1,\ y=1$

$$f(x) = \frac{(x-1)^3(x-3)^2}{x^2(x+3)(x-3)}$$

$$= \frac{(x-1)^2(x-3)^2}{x^2(x+3)}$$

Vertical Asymptotes at $x=0,\ x=-3$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{(x-1)^3(x-3)^2}{x^2(x+3)} = \lim_{x\to\infty} \frac{\left(x\left(1-\frac{1}{x}\right)\right)^3\left(x\left(1-\frac{3}{x}\right)\right)^2}{x^2(x)\left(1+\frac{3}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{x^3\left(1-\frac{1}{x}\right)^3 x^2\left(1-\frac{3}{x}\right)}{x^3\left(1+\frac{3}{x}\right)} = \lim_{x\to\infty} \frac{x^2\left(1-\frac{1}{x}\right)^3\left(1-\frac{3}{x}\right)}{\left(1+\frac{3}{x}\right)} = \infty$$

✓ Also $\lim_{x\to-\infty} f(x) = \infty$

11. If after t seconds the displacement of an object moving along a straight line is $s(t) = 3t^2 - 20t$ meters, find the (instantaneous) velocity at $t = 3$.

$$v(t) = s'(t) = 6t - 20$$

$$v(3) = s'(3) = 18 - 20 = -2$$

A. -6 m/sec

[✓ **B.** -2 m/sec]

C. 0 m/sec

D. 18 m/sec

E. 48 m/sec

12. If $f(x) = \left(2 + \frac{3}{x}\right)$, then the expression $\dfrac{f(a+h) - f(a-h)}{2h} =$

$$\frac{f(a+h) - f(a-h)}{2h}$$

$$= \frac{\left(\cancel{2} + \frac{3}{a+h}\right) - \left(\cancel{2} + \frac{3}{a-h}\right)}{2h}$$

$$= \frac{\frac{3}{a+h} - \frac{3}{a-h}}{2h} = \frac{\frac{3(a-h) - 3(a+h)}{a^2 - h^2}}{2h}$$

$$= \frac{\cancel{3a} - 3h - \cancel{3a} - 3h}{(a^2 - h^2)(2h)} = \frac{-3}{a^2 - h^2}$$

A. $1 + \dfrac{3}{a^2 - h^2}$

B. $2 + \dfrac{6}{a^2 - h^2}$

C. $\dfrac{1}{h} - \dfrac{1}{2(a^2 - h^2)}$

[✓ **D.** $-\dfrac{3}{a^2 - h^2}$]

E. $-\dfrac{6}{a^2 - h^2}$