SOLUTIONS

GREEN

1. Find the derivative of $y = \dfrac{\sin 3x}{3x}$.

$$\frac{d}{dx}\left\{\frac{\sin(3x)}{(3x)}\right\} = \frac{(3x)\{3\cos 3x\} - (\sin 3x)3}{9x^2}$$

$$= \frac{3x\cos 3x - \sin 3x}{3x^2}$$

$\boxed{\text{A.}}$ $y' = \dfrac{3x\cos 3x - \sin 3x}{3x^2}$

B. $y' = \dfrac{x\cos 3x - \sin 3x}{3x^2}$

C. $y' = \dfrac{3x\cos 3x + \sin 3x}{x^2}$

D. $y' = \dfrac{x\cos x - \sin x}{x^2}$

E. $y' = \dfrac{\sin 3x - 9x\cos 3x}{3x^2}$

2. $\displaystyle\lim_{x\to 0} \frac{\tan 2\pi x}{\sin 4x} = \lim_{x\to 0} \frac{\sin 2\pi x}{\cos 2\pi x} \cdot \frac{1}{\sin 4x}$

$$= \lim_{x\to 0}\left(\frac{\sin 2\pi x}{2\pi x}\right)\left(\frac{1}{\cos 2\pi x}\right)\left(\frac{4x}{\sin 4x}\right)\left(\frac{2\pi x}{4x}\right)$$

$$= (1)(1)(1)\left(\frac{2\pi}{4}\right)$$

$$= \frac{\pi}{2}$$

A. 0

B. $\frac{1}{2}$

C. $\frac{\pi}{4}$

$\boxed{\text{D}}$ $\frac{\pi}{2}$

E. 2π

II L'Hôpital's Rule $\Rightarrow$ $\displaystyle\lim_{x\to 0}\frac{\tan 2\pi x}{\sin 4x} = \lim_{x\to 0}\frac{2\pi \sec^2(2\pi x)}{4\cos 4x}$

$$= \frac{2\pi(1)}{4(1)} = \frac{\pi}{2} \quad \text{II}$$

3. Find an equation of the tangent line to the curve $\ln(xy) = 2x^2 - y - 1$ at the point $(1,1)$.

$\ln x + \ln y = 2x^2 - y - 1$

$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 4x - \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{4x - \frac{1}{x}}{\left(\frac{1}{y}+1\right)}\Bigg|_{(1,1)} = \frac{3}{2}$

$\therefore \quad y - 1 = \frac{3}{2}(x-1)$

$y = 1 + \frac{3}{2}x - \frac{3}{2} = \frac{3}{2}x - \frac{1}{2}$

A. $y = \frac{1}{2}x + \frac{1}{2}$

$\boxed{\textbf{B.}}$ $y = \frac{3}{2}x - \frac{1}{2}$

C. $y = -\frac{1}{2}x + \frac{3}{2}$

D. $y = \frac{3}{2}x + \frac{3}{2}$

E. $y = x$

4. If $f(x) = x^2 + 2^{x^2}$, compute $f'(-1)$.

$f'(x) = 2x + 2^{x^2}(\ln 2)(2x)$

$f'(-1) = 2(-1) + 2^{(-1)^2}(\ln 2)(2)(-1)$

$= -2 + 2(\ln 2)(-2)$

$= -2 - 4\ln 2$

A. $-2+2\ln 2$

B. $-2-2\ln 2$

$\boxed{\textbf{C.}}$ $-2-4\ln 2$

D. $-2-8\ln 2$

E. -4

5. If $y = \sin(3x^2+1)$, find y''.

$y' = 6x\cos(3x^2+1)$

$y'' = 6x\{-6x\sin(3x^2+1)\} + 6\cos(3x^2+1)$

$= -36x^2\sin(3x^2+1) + 6\cos(3x^2+1)$

A. $3\cos(3x^2+1) - 9x^2\sin(3x^2+1)$

B. $6\cos(3x^2+1) - 4x^2\sin(3x^2+1)$

C. $3\cos(3x^2+1) + 9x^2\sin(3x^2+1)$

$\boxed{\textbf{D.}}$ $6\cos(3x^2+1) - 36x^2\sin(3x^2+1)$

E. $-6x\cos(3x^2+1)$

6. $\frac{d}{dx}\left\{\tan^{-1}\left(\frac{2}{x^2}\right)\right\} = ?$

$$= \frac{1}{1+\left(\frac{2}{x^2}\right)^2}\frac{d}{dx}\left\{\frac{2}{x^2}\right\}$$

$$= \frac{1}{\left(1+\frac{4}{x^4}\right)}\left\{\frac{-4}{x^3}\right\}$$

$$= \left(\frac{x^4}{x^4+4}\right)\left(\frac{-4}{x^3}\right) = -\frac{4x}{x^4+4}$$

A. $\frac{2}{1+x^2}$

$\boxed{\textbf{B.}}$ $-\frac{4x}{x^4+4}$

C. $\frac{2x^3}{x^4+4}$

D. $-\frac{4x^3}{x^4+4}$

E. $\frac{4x^3}{x^4+4}$

7. An arrow is shot directly upward so that after t seconds its height in meters is given by

$$s(t) = v_0\,t - \frac{1}{2}\,g\,t^2$$

where v_0 and g are constants. What is the maximum height the arrow attains?

Max height when $s'(t) = 0$

$v_0 - gt = 0$

$t = \frac{v_0}{g}$

Max height is $s\left(\frac{v_0}{g}\right) = v_0\left(\frac{v_0}{g}\right) - \frac{1}{2}g\left(\frac{v_0}{g}\right)^2$

$= \frac{v_0^2}{g} - \frac{v_0^2}{2g} = \frac{v_0^2}{2g}$

A. $\frac{v_0^2}{g}$ meters

B. $\frac{v_0}{2g}$ meters

$\boxed{\textbf{C.}}$ $\frac{v_0^2}{2g}$ meters

D. $\frac{2v_0}{g}$ meters

E. $v_0\,g$ meters

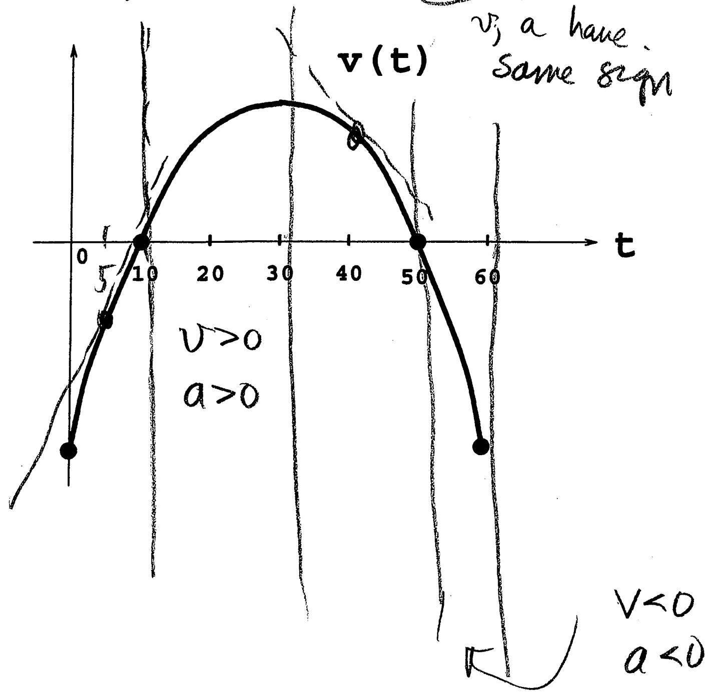
8. Given that the graph of the velocity $v(t)$ of an object is shown below for $0 \le t \le 60$, which of the following statements are **TRUE** ?

T **(I)** $a(5) > 0$ (the acceleration is positive at $t = 5$)

F **(II)** $a(40) > 0$ (the acceleration is positive at $t = 40$)

T **(III)** The object is *speeding up* when $10 < t < 30$ and $50 < t < 60$.

v, a have same sign



A. Only (I)

$\boxed{\textbf{B.}}$ Only (I) and (III)

C. Only (I) and (II)

D. Only (II) and (III)

E. Only (III)

9. $\dfrac{d}{dx}\left\{5\sinh(\ln x)\right\} =$

A. $\frac{5}{2}\left(1-\frac{1}{x^2}\right)$

B. $\frac{1}{2}\left(x+\frac{1}{x}\right)$

C. $\frac{5}{4}\left(1+\frac{1}{x^2}\right)$

$\boxed{\text{D.}}$ $\frac{5}{2}\left(1+\frac{1}{x^2}\right)$

E. $\frac{5}{2}\left(x-\frac{1}{x}\right)$

Soln1: $5\sinh(\ln x) = 5\left[\dfrac{e^{\ln x}-e^{-\ln x}}{2}\right] = 5\left[\dfrac{x-\frac{1}{x}}{2}\right]$

$= \frac{5}{2}\left(x-\frac{1}{x}\right) \quad \therefore \{5\sinh(\ln x)\}' = \frac{5}{2}\left(1+\frac{1}{x^2}\right)$

Soln2: $\{5\sinh(\ln x)\}' = \{5\cosh(\ln x)\}\left(\frac{1}{x}\right)$

$= 5\left\{\dfrac{e^{\ln x}+e^{-\ln x}}{2}\right\}\left(\frac{1}{x}\right)$

$= 5\left\{\dfrac{x+\frac{1}{x}}{2}\right\}\left(\frac{1}{x}\right) = \dfrac{5}{2x}\left(x+\frac{1}{x}\right) = \frac{5}{2}\left(1+\frac{1}{x^2}\right)$

10. If $f'(x) = \dfrac{2x}{x^2+1}$ and $H(x) = f\left(\sqrt{x}\right)$, then find $H'(4)$.

A. 1

B. $\frac{2}{5}$

C. $\frac{3}{5}$

D. $\frac{1}{3}$

$\boxed{\text{E.}}$ $\frac{1}{5}$

$H(x) = f(\sqrt{x})$, use Chain Rule

$\Rightarrow H'(x) = f'(\sqrt{x})\left(\dfrac{1}{2\sqrt{x}}\right)$

Now $f'(x) = \dfrac{2x}{x^2+1}$

$\therefore f'(\sqrt{x}) = \dfrac{2\sqrt{x}}{(\sqrt{x})^2+1} = \dfrac{2\sqrt{x}}{x+1}$

$\therefore H'(x) = \dfrac{2\sqrt{x}}{x+1}\left(\dfrac{1}{2\sqrt{x}}\right) = \dfrac{1}{x+1}$

$H'(4) = \dfrac{1}{4+1} = \dfrac{1}{5}$

11. The population $p(t)$ of a certain bacteria grows at a rate proportional to its size and hence $p(t) = p(0)e^{kt}$. The table below shows data collected by a lab assistant. At what time t will there be 30 bacteria present?

t (hours)	$p(t)$
0	10
8	20

$p(t) = 10e^{kt}$

$20 = p(8) = 10\,e^{k(8)}$

$2 = e^{k(8)}$

$\ln 2 = k(8)$

$\frac{1}{8}\ln 2 = k$

$\therefore\ p(t) = 10\,e^{\frac{t}{8}\ln 2}$

$p(t) = 30$ when

$10\,e^{\frac{t}{8}\ln 2} = 30$

$e^{\frac{t}{8}\ln 2} = 3$

$\frac{t}{8}\ln 2 = \ln 3 \Rightarrow t = 8\,\frac{\ln 3}{\ln 2}$

A. $t = 10\left(\frac{\ln 3}{\ln 2}\right)$

B. $t = 8\ln\left(\frac{2}{3}\right)$

C. $t = 10\left(\frac{\ln 2}{\ln 3}\right)$

$\boxed{\textbf{D.}}$ $t = 8\left(\frac{\ln 3}{\ln 2}\right)$

E. $t = 10\ln\left(\frac{2}{3}\right)$

12. A particle moves along the hyperbola $\frac{y^2}{2} - x^2 = 1$. As it reaches the point $(-1, 2)$, the y-coordinate is increasing at a rate of 3 in/sec. How fast is the x-coordinate of the point changing at that instant?

given rate: $\frac{dy}{dt} = 3$ in/sec

desired rate: $\frac{dx}{dt}$ @ $(-1,2)$.

Relation: $\frac{y^2}{2} - x^2 = 1$

$\Rightarrow \frac{d}{dt}\left\{\frac{y^2}{2} - x^2\right\} = \frac{d}{dt}\{1\}$

$\frac{1}{2}(2y)\frac{dy}{dt} - (2x)\frac{dx}{dt} = 0$

$\frac{dx}{dt} = \frac{y}{2x}\frac{dx}{dt} = \frac{(2)}{2(-1)}(3) = -3$ in/sec

A. $-\frac{2}{3}$ in/sec

$\boxed{\textbf{B.}}$ -3 in/sec

C. -2 in/sec

D. 1 in/sec

E. $-\frac{3}{2}$ in/sec