Homework Set # 7

1. Page 87 :  # 13(c) .

2. Construct examples to show that

   (a) $\log(z_1 z_2) \neq \log z_1 + \log z_2$

   (b) $\log \left( \frac{1}{z_3} \right) \neq -\log z_3$ .

3. If $z \neq \pm i$, show that $w = \tan^{-1} z = \frac{i}{2} \log \left( \frac{i+z}{i-z} \right)$.

   Use this to solve the equation $\tan z = -2 + i$.

4. If $f(z) = i^z$, where the principal value is chosen, compute $f'(1)$.

5. Let $C_R$ be the semicircle of radius $R$ in the upper half-plane, where $R > 1$ as shown below, and let $f(z) = \frac{ze^{2iz}}{z^2 + 1}$ and $g(z) = \frac{\log z}{z^2 + 1}$, show that for all $z \in C_R$, we get the estimates:

   $$|f(z)| \leq \frac{R}{R^2 - 1} \quad \text{and} \quad |g(z)| \leq \frac{\log R + \pi}{R^2 - 1} .$$