Homework Set # 9


2. Compute \( I = \int_C \frac{e^{\pi z}}{z^2(z^2 + 1)} \, dz \) using Partial Fractions and using the Generalized Cauchy Theorem, where \( C \) is the contour:

![Contour Diagram]

3. If \( C \) is the circle \(|z - 3i| = 4\) traversed once in a positive sense, compute these integrals:

(a) \( \int_C \frac{e^{\pi z}}{z^4 + 16z^2} \, dz \)

(b) \( \int_C \frac{\cos z}{(z + 1)^4} \, dz \)

4. Show that \( \int_{|z|=R} e^{(\frac{1}{z})} \sin \left(\frac{1}{z}\right) \, dz = 2\pi i \), where the circle \(|z| = R\) is traversed in a positive sense and \( R > 0 \). (Hint: change variables, let \( w = \frac{1}{z} \). What happens to the circle \(|z| = R \)?)


**Extra Credit Problem:** Let \( D \) be the doubly-connected domain bounded by simple closed contours \( \Gamma \) and \( \gamma \) as shown below. If \( f \) is analytic in \( D \) and on the boundary of \( D \), show that

\[
\frac{1}{2\pi i} \int_{\Gamma} \frac{f(z)}{z - z_0} \, dz = f(z_0) + \frac{1}{2\pi i} \int_{\gamma} \frac{f(z)}{z - z_0} \, dz
\]

for any \( z_0 \) inside \( D \):

![Doubly-Connected Domain Diagram]

(This is sometimes called the Cauchy Integral Formula For Doubly-Connected Domains.)