Homework Set # 8

1. Find the center of mass of the region $D$ bounded by $y = x^2$ and $y = 1$ if the density at $(x, y)$ is given by $\rho(x, y) = x + 1$:

![Diagram of region D]

2. Show that the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by $A = \pi ab$ $(a, b > 0)$:

\[ \begin{align*}
  &\text{Hint: Compute } \int \int_D dx \, dy \text{ by changing variables using the mapping } T : \\
  &\quad \begin{cases}
    \frac{x}{a} = r \cos \theta \\
    \frac{y}{b} = r \sin \theta
  \end{cases}
\end{align*} \]

3. Compute $I = \int \int_D \frac{(y + x)}{xy} \, dx \, dy$, where $D$ is the region:

\[ \begin{align*}
  &\text{Hint: Let } u = xy \text{ and } v = y - x \text{ and recall that } \frac{\partial(x, y)}{\partial(u, v)} \frac{\partial(u, v)}{\partial(x, y)} = 1
\end{align*} \]

4. Page 400 : # 3(b).

5. Find the mass of a wire bent in the shape of a helix $\vec{c}(t) = (\cos 2t, \sin 2t, t)$, for $0 \leq t \leq \frac{\pi}{2}$, and whose density is $\rho(x, y, z) = 8y$.


7. If $C$ is the curve $y = x^2 + 2x$ from $(0, 0)$ to $(1, 3)$, compute the following:

\[ \begin{align*}
  &\int_C (x^2 - y - 2) \, ds, \quad \int_C (xy + 1)\vec{i} - x\vec{j} \cdot d\vec{s} \quad \text{and} \quad \int_C 2xy \, dx + x \, dy.
\end{align*} \]